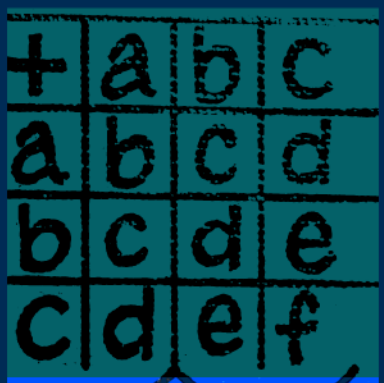
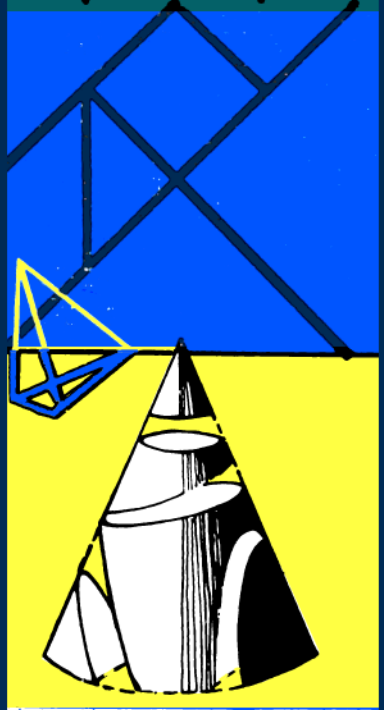
Take Shapes Lines and Letters

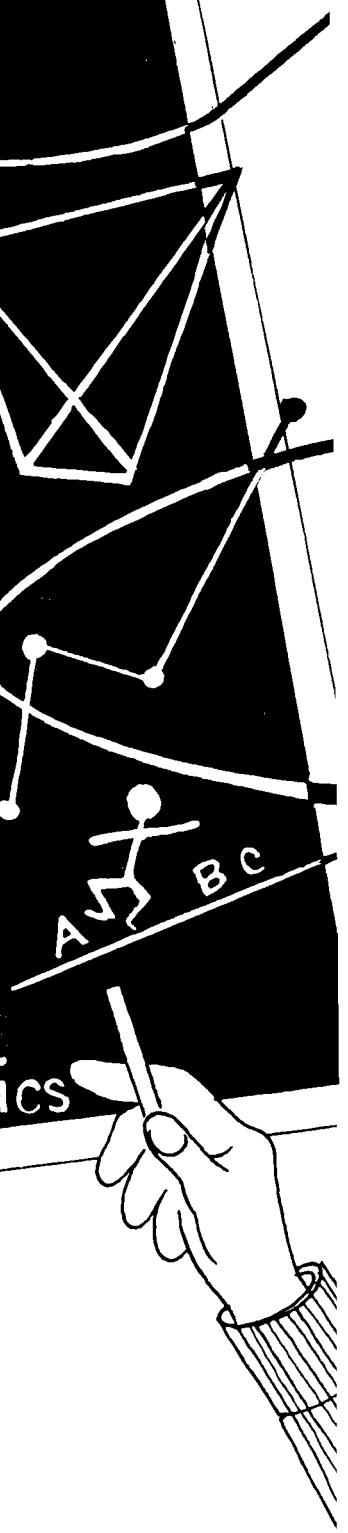




Jeanne Bendick
Marcia Levin

Take Shapes, Lines and Letters

New Horizons in Mathematics



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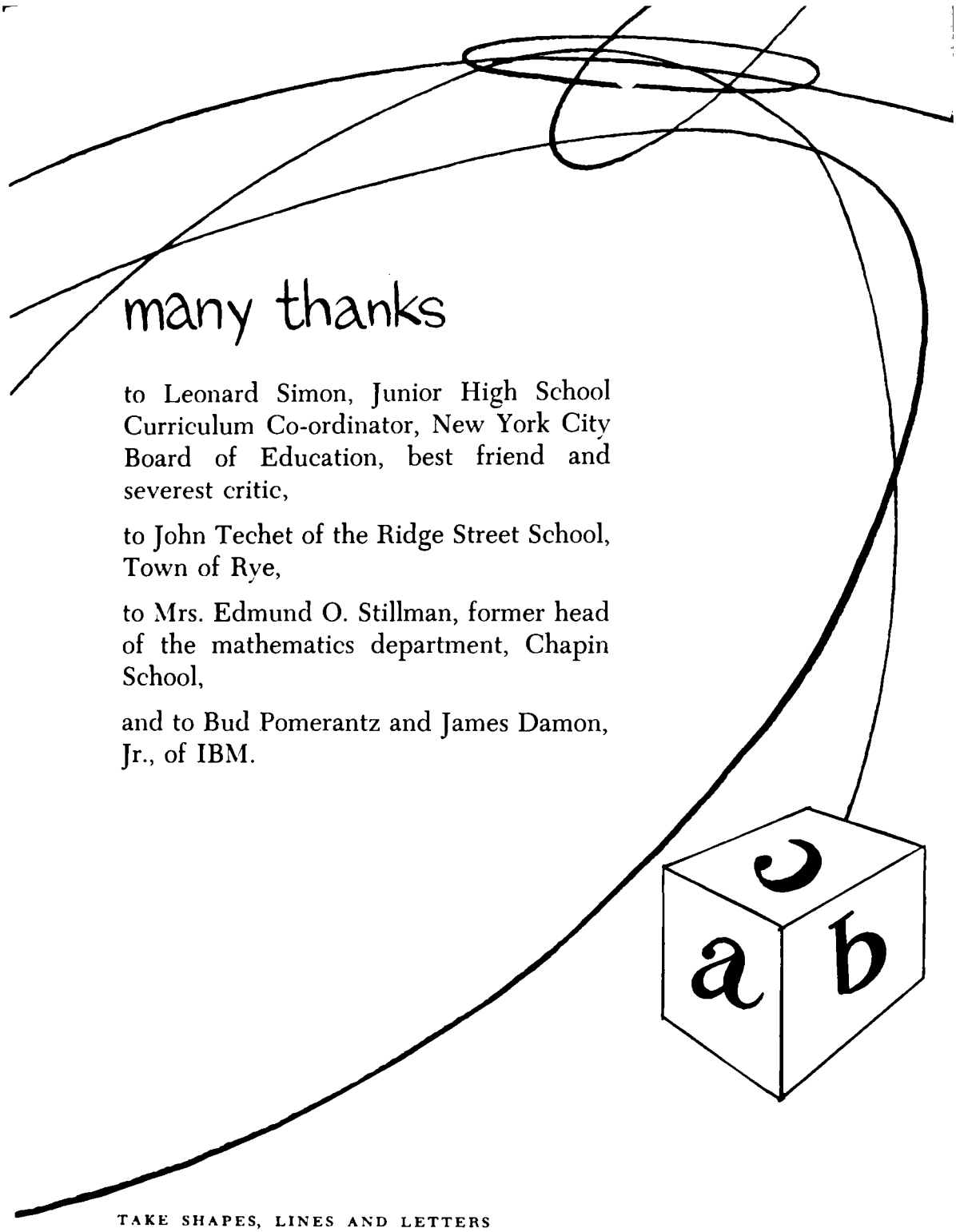
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TAKE SHAPES, LINES AND LETTERS

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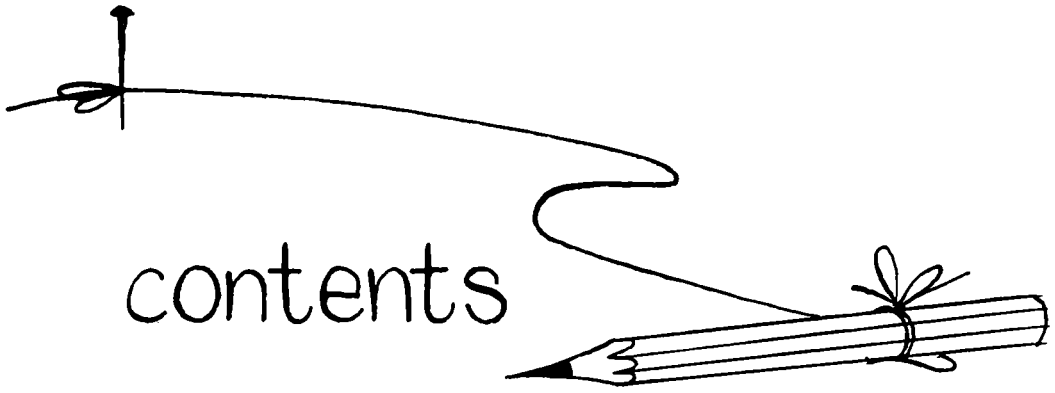

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You can do anything in this book with

a sharp pencil

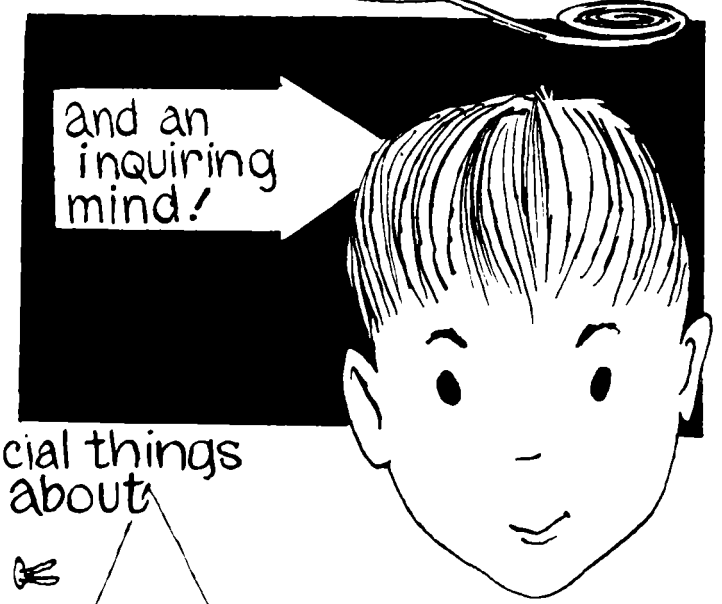
paper

pins

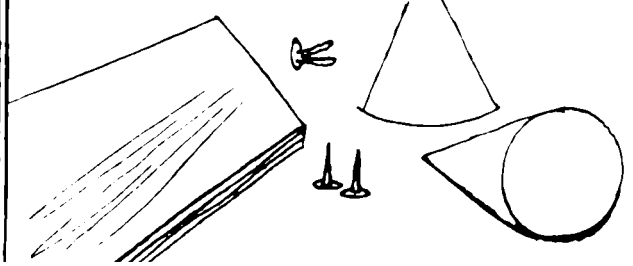
a compass and scissors

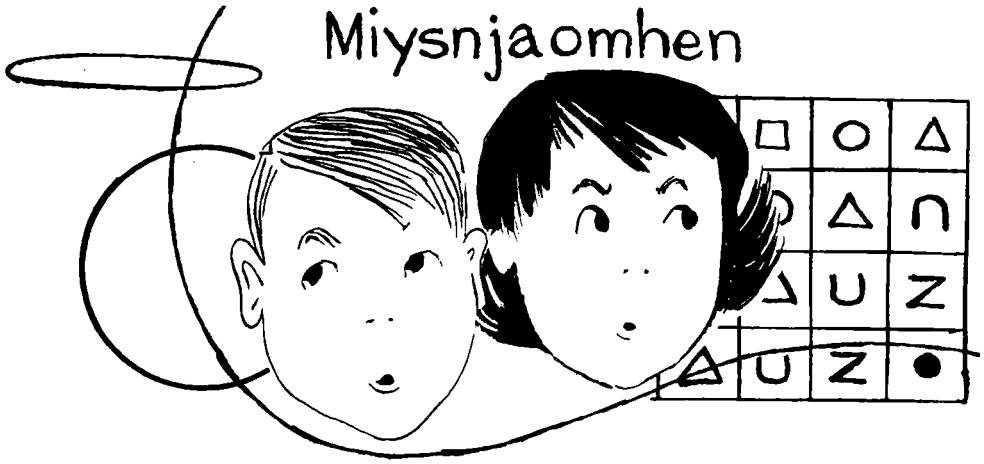
string

ruler



a few special things we'll talk about





SHAPES, LINES AND LETTERS

Are shapes, lines and letters mathematics? Where are the numbers?

You don't always need numbers in mathematics.

But isn't mathematics the study of numbers?

Well, yes and no.

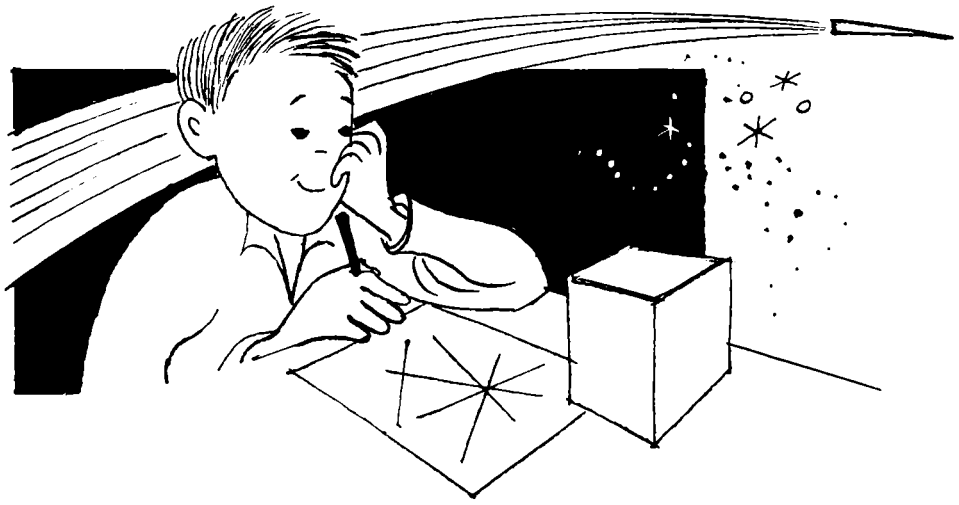
The word, *mathematics* comes from two ancient Greek words, *manthano,* which means "to learn," and *mathema,* which means "science." Nothing about numbers or counting!

Mathematics is a kind of learning or thinking. Many people call it the Queen of Sciences, because almost every other kind of scientific knowledge depends on it. The study of numbers is only one part of mathematics. It is also the study of other ideas—

ideas about lines and points, shapes and planes,

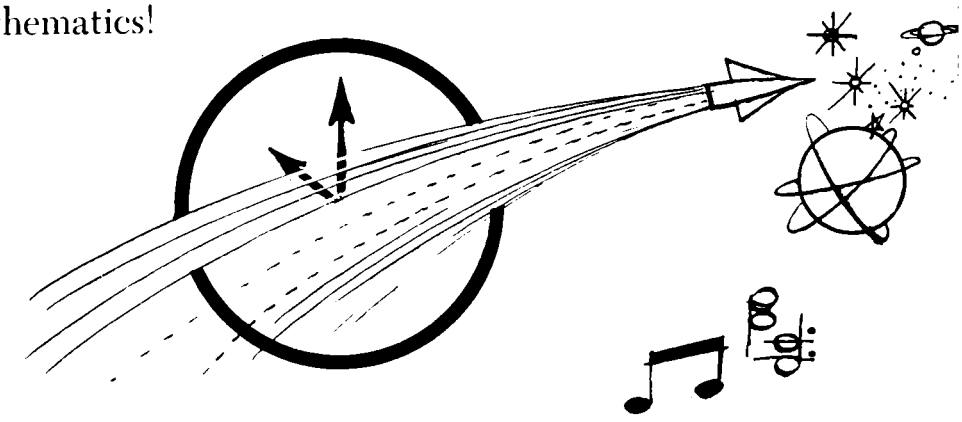
the way you show them on paper,

and the way you think about them, which is different.

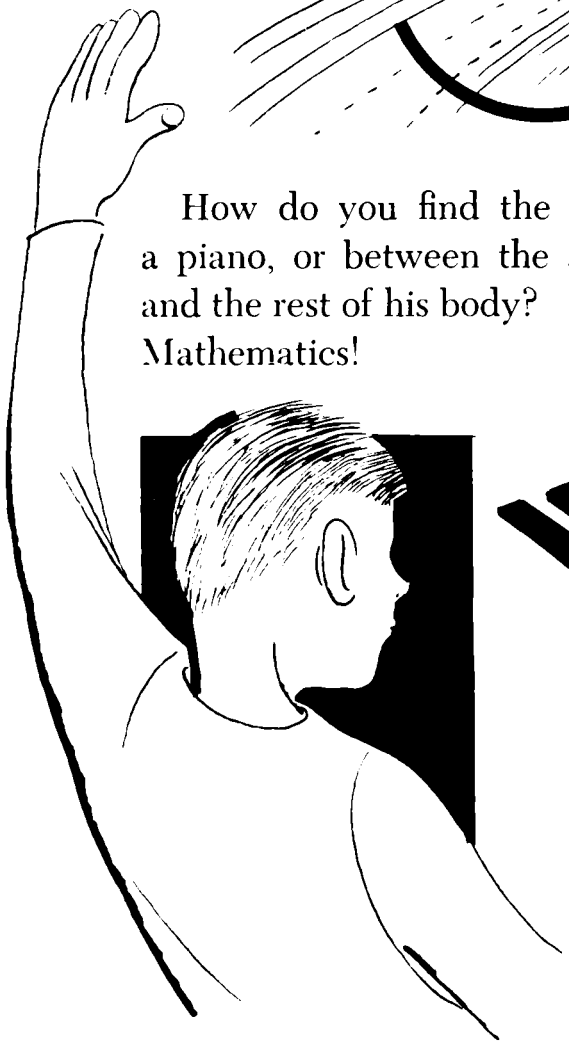


Mathematics is the study of relationships, too.

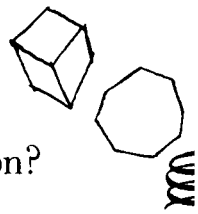
How can you find the relationship between how fast you travel, how long you travel and how far you go?
Mathematics!



How do you find the relationship between the notes on a piano, or between the size of a man's head or his arms—and the rest of his body?
Mathematics!



Mathematics tells us about sizes and shapes.



What is a cube?

What is an octagon?

What is a helix? (hé liks)
Mathematics describes them.

Of course sometimes we use numbers when we talk about shapes or relationships. But even if we do, the numbers aren't the most important thing.

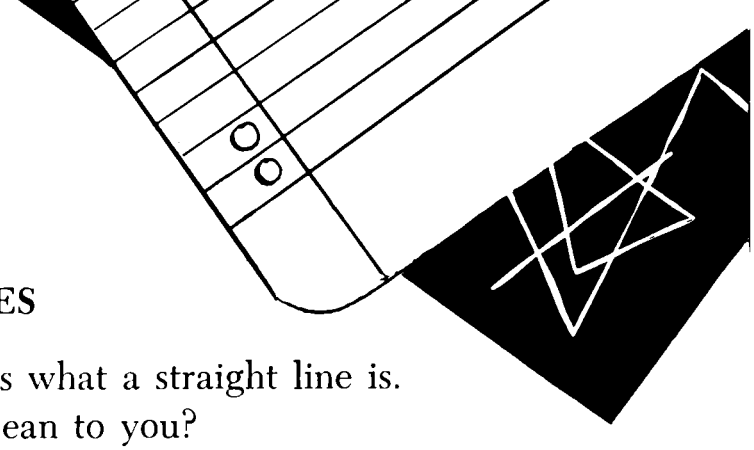
Mathematics can help us to decide how to show a straight line or a circle, what the orbit of a satellite or the path of a rocket will look like, how to make a secret code or figure out what part of your time you spend watching television. It can even show you how to take off your vest without taking off your coat! And you won't need numbers.

There are really two worlds in mathematics.

One is the physical world—the world around you, full of things you can see and touch; lines you can draw; numbers or letters you can write; rulers for measuring.

The other is a world which is made up of ideas. There is nothing you can see or touch in that world. You can only imagine it. You can't make a point with a pencil in that world. The lines you measure or the points you draw are part of the real world. They only *represent* the ideas in the world of mathematics.

You have to go from one world to the other in your mathematics of shapes, lines and letters, but your mind is used to that. You do it a hundred times every day!



LINES AND STRAIGHT LINES

Everyone knows what a straight line is.

What does it mean to you?

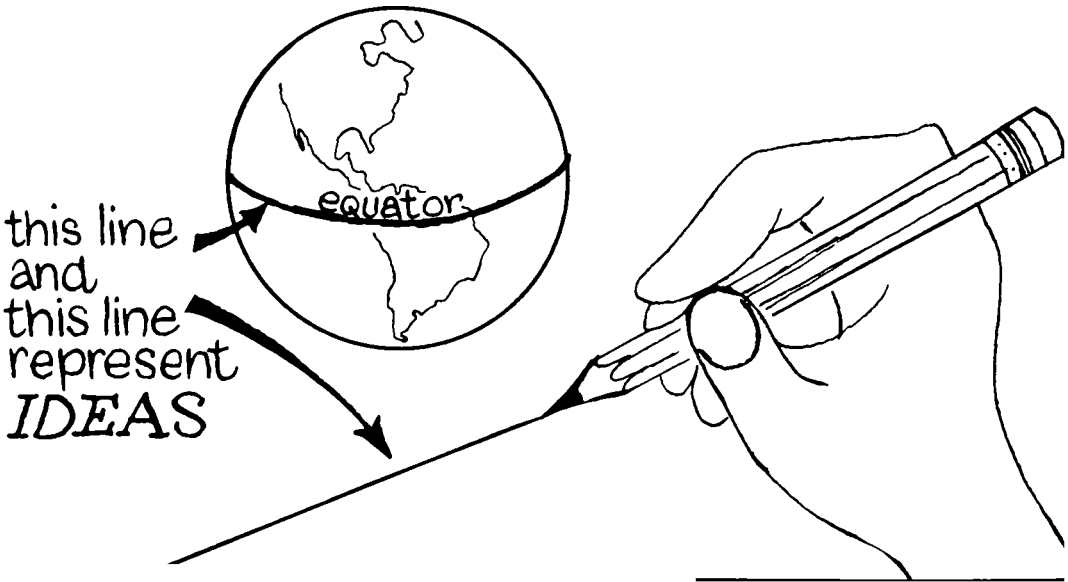
In the physical world around you, you could point to the lines on your notebook paper. The edges of books and tables and rooms are straight lines. So are the lines that mark off tennis courts and football fields and baseball diamonds.

But in the world of geometry, in the mathematics in your mind, a straight line is an idea. You can't draw it on a paper or a tennis court. You can't erase it.

Why not?

The line drawn on the paper only marks the *idea* of a line. It is not the line itself. You have heard about lines like this before, haven't you?

The equator is an *imaginary* line that separates the northern hemisphere from the southern hemisphere. Everybody accepts the *idea* of the equator, but there is no real line around the center of the earth. If you go from one state to another, or one country to another, no matter how hard you search, you won't find the line that marks the boundary. But you can see it on any map.



Can you begin to think about imaginary lines now?

Well, not yet. First we have to think about points.

What is a point? In the world of mathematicians, a point is an idea. To mathematicians, a point has no dimensions. It has no length, no width, no height. In our everyday world we represent this idea with a dot on a piece of paper.

•

Close your eyes. Can you imagine space filled with points? (You need imagination in mathematics, you know.)

Imagine an unending set of points in space.

Take your paper and pencil and draw some dots to represent points.

• • •• • • •

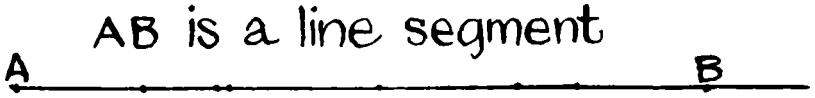
Make some of them close together, like this

.....

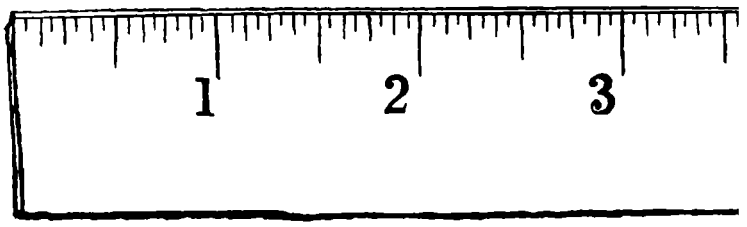
What do they look like now? You can see that if they are close enough, a *series of points* will form a *line.* That's what a line is—a series of points that goes on endlessly.

The line we draw—the line in the physical world—might start at a point we call A, and stop at a point we call B.

But the line in the mathematical world, the idea of a line, doesn't do that. It goes on and on, extending endlessly in both directions. What we drew, the part of the line containing points A and B, and any points between them, is called a *line segment.* The line segment has one dimension, length.

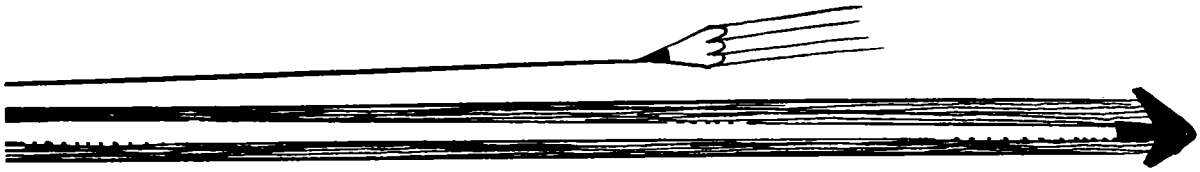


you can measure the length of a line segment



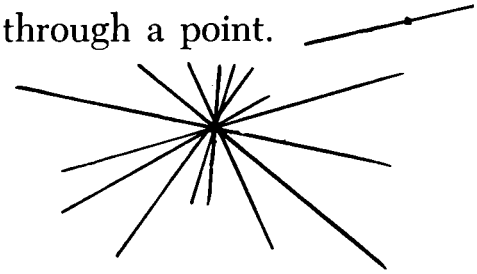
Some people say that a line is formed as a point moves.

If you took a point, and moved it from one part of the paper to another, which is what you do when you move the pencil across the paper, then wouldn't you have a line?



It's something like the vapor trail behind a jet plane, which shows where the plane has been.

We can always imagine a line through a point.



In fact, an unlimited number of lines can pass through any point.

Now imagine two points.

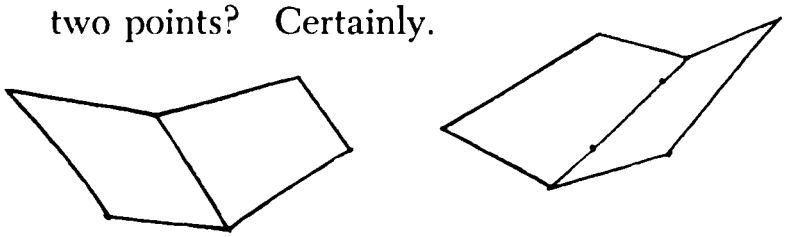
Draw dots to represent them.

How would you measure the shortest distance between those two points? You can see right away that you would have to measure it along a straight line. If you made the line curve or wiggle, it would be longer.

If you have two points there is only and exactly one straight line that will connect them. So we say, a straight line is *determined* by two points.

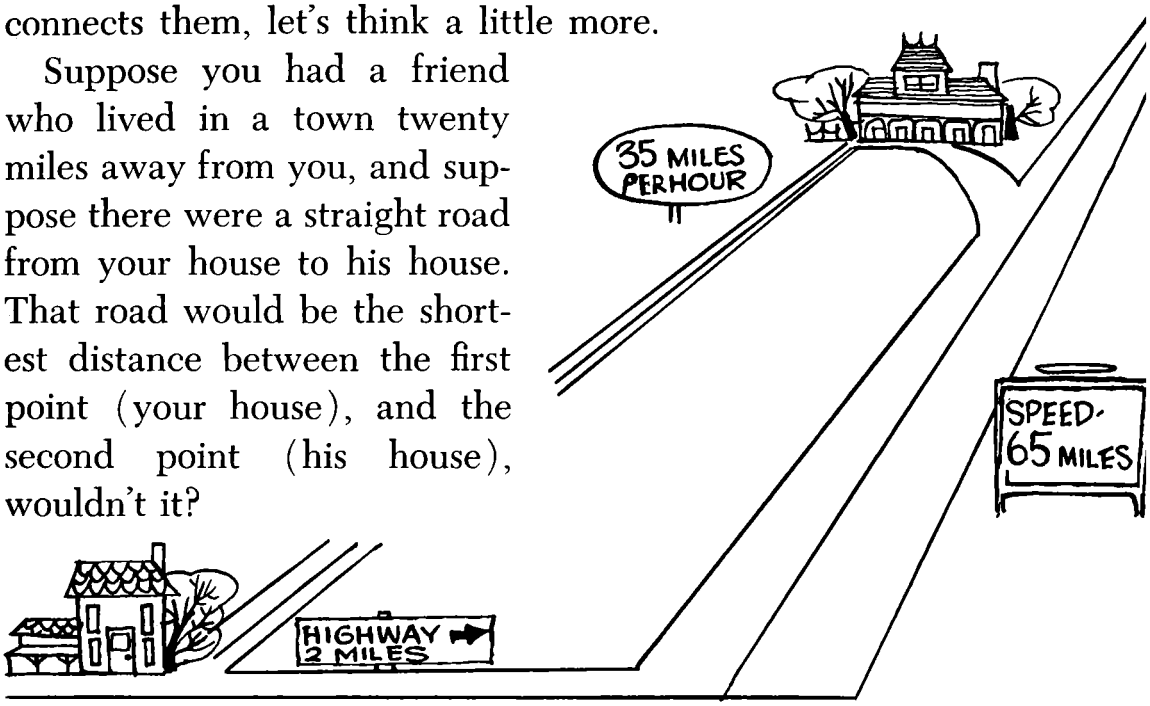
It's easy to draw a straight line if you have a ruler for your pencil to follow. But do you know an even easier way?

Take a piece of paper, any size or shape, and fold it. When you open the paper you will see that the fold is a straight line. Can you make a fold which passes through two points? Certainly.



Now that we have agreed that the shortest distance between two points is measured along the straight line that connects them, let's think a little more.

Suppose you had a friend who lived in a town twenty miles away from you, and suppose there were a straight road from your house to his house. That road would be the shortest distance between the first point (your house), and the second point (his house), wouldn't it?

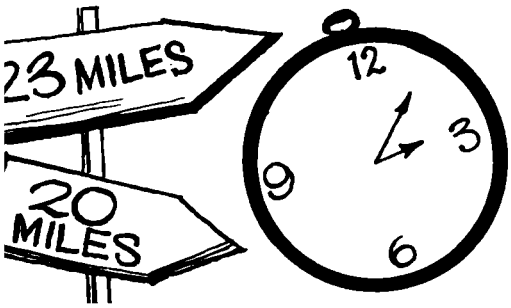


Now suppose a superhighway were built two miles away from your house, with an exit one mile from your friend's house. The old road has a 35-mile speed limit, but you can drive 65 miles an hour on the new road.

Even though it's three miles longer you'd probably take the new road, because while it is longer in *distance,* it is shorter in *time.*

Thousands and hundreds and even fifty years ago, when there were no superhighways or jet planes or rockets, time was not important in mathematics, and nobody had to worry about it. But in our fast-moving world, time is very important.

Now when we think about the shortest distance between two points we must think



Is it the shortest in length?
Is it the shortest in time?

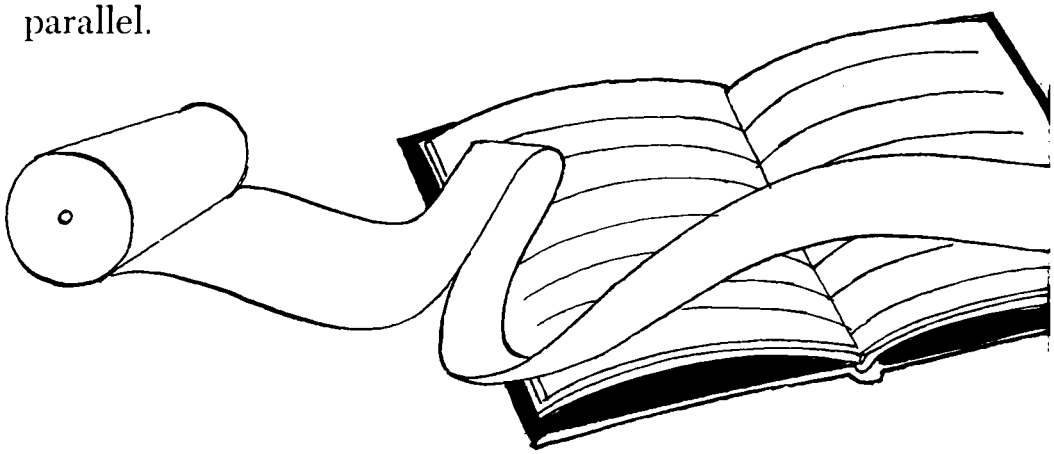
PARALLEL LINES

A line extends in either direction forever.

If we imagine two lines that always stay the same distance apart we call them *parallel* lines.

Railroad tracks represent parallel lines. If you traveled the tracks from one end to the other you would see that they never get any closer together or any farther apart.

Have you ever seen a roll of adding-machine tape at the checkout counter of a supermarket? If you unrolled that whole roll of tape you would see that the edges are always parallel.

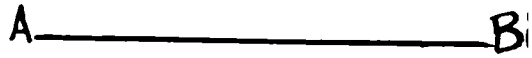


The pages of this book have parallel sides. The lines of type are parallel too.

Mathematicians have a shorthand way of writing "is parallel to."

They use this symbol: ||

If we have a line (call it AB)

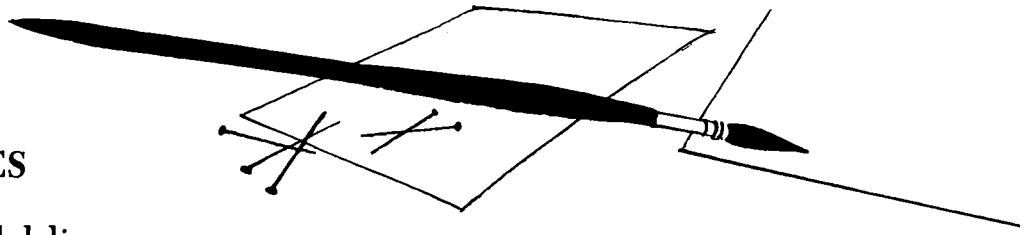


and we have another line (call it CD)

and they are always the same distance apart, we can say, "AB is parallel to CD."

Mathematicians write it this way: AB || CD

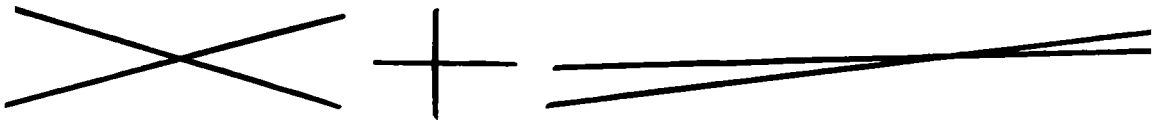
Isn't that quick and easy?



ANGLES

Parallel lines never cross.

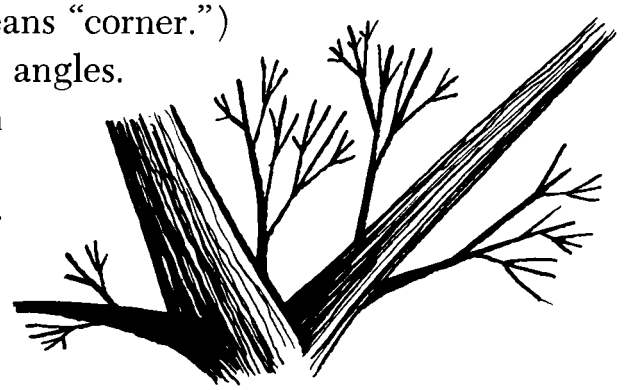
When lines do cross, we say they *intersect.* The place where two streets cross is an intersection, isn't it? Two straight lines can intersect at only one point.



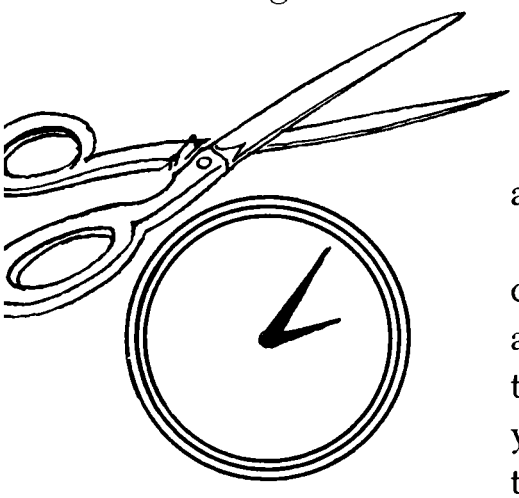
Lines which intersect form *angles.* (*Angle* comes from the Latin word *angulus,* which means "corner.")

Wherever you look, you see angles.

Tree branches often form angles.



The blades of an open scissors form an angle.

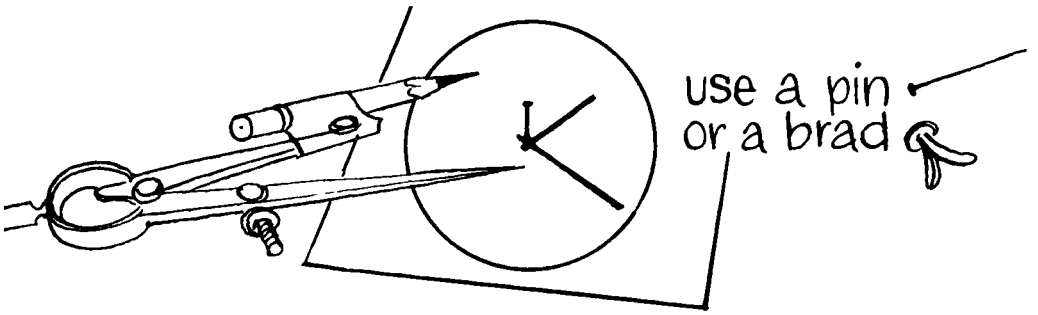


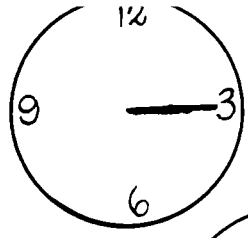
The hands on a clock form an angle.

If you watch a clock, you can see some interesting things about angles. You can see them even better if you make your own special clock, like this:

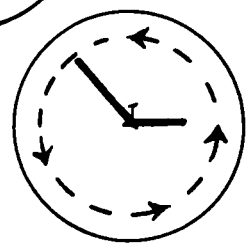
With your compass, draw a circle on a piece of cardboard.

Cut two thin clock hands out of cardboard. Fasten them at the center of the circle with a brad, or just stick a pin through them to the center.



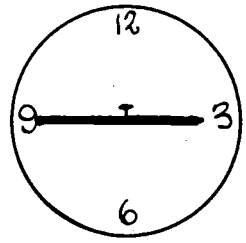


Start both hands at the "3" position.



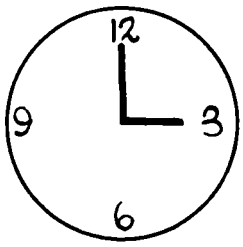
Now move one hand backward (counterclockwise), around the clock, like this

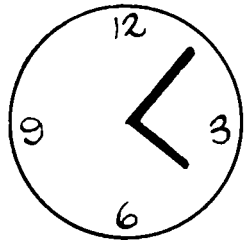
When it has made one complete turn around the clock, mathematicians say that it has made one *revolution.* In every position, except directly on top of each other, the hands formed an angle.

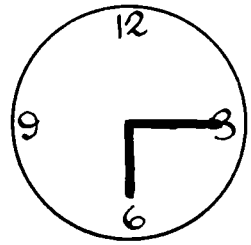


Even when they are straight out, like this—when one hand has made a half-revolution—they are called a *straight angle.*

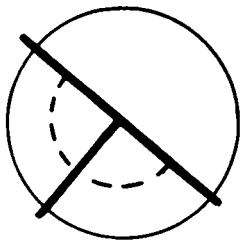
When one hand is pointing to 12 and the other to 3, the angle formed is a quarter-revolution, or a *right angle.* Of course the hands don't have to be on these particular numbers to form a right angle. Any quarter-revolution will do.

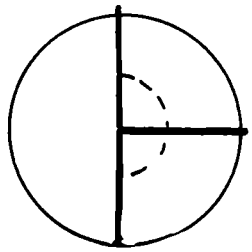


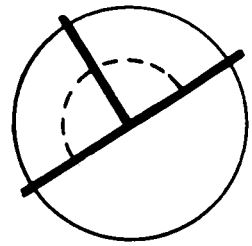




Lines which make right angles when they meet are called *perpendicular* (pur' pen dik' ye ler) lines. Perpendicular lines are always at right angles to each other. And the angles on each side are always exactly the same, each a quarter-revolution, like this

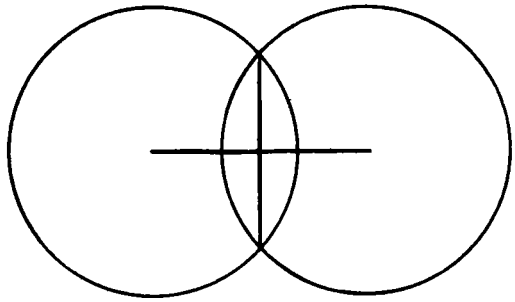






Here is a way to make right angles with a compass and ruler.

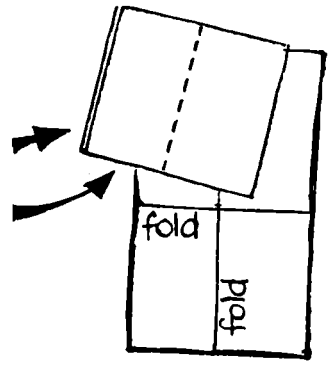
Draw two overlapping circles of equal size, like this



Connect the centers with a straight line.

Connect the points where the two circles intersect with a straight line. And there are four right angles.

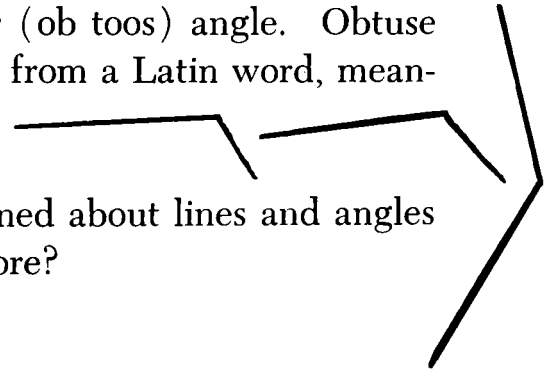
It is even easier to make right angles by folding a piece of paper, first like this, so the opposite edges meet exactly, then again so the other edges meet. When you unfold the paper you will have two lines at right angles to each other.



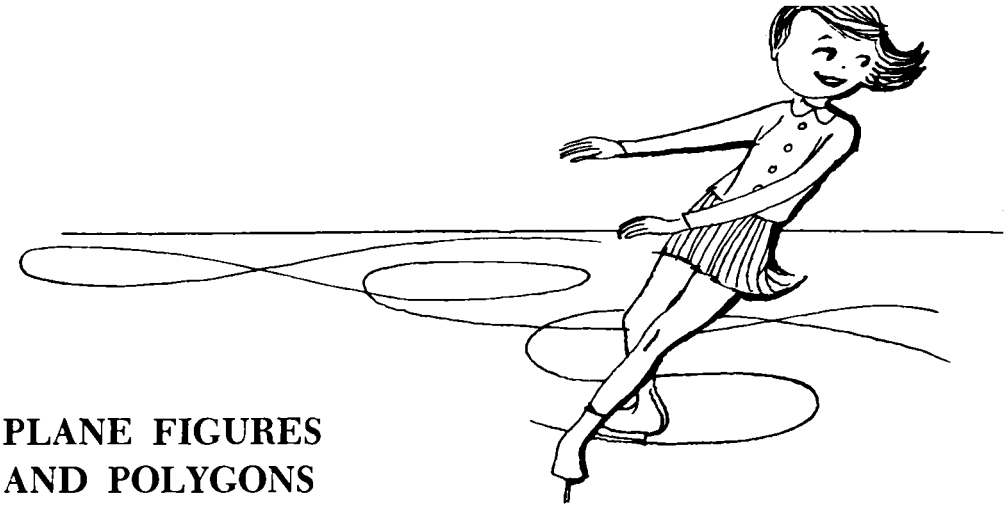
An angle which is smaller than a right angle is called an *acute* angle. Acute means "sharp." Can you see why these are acute angles?

An angle that is larger than a right angle but smaller than a straight angle is called an *obtuse* (ob toos) angle. Obtuse is the opposite of acute, and comes from a Latin word, meaning "dull."

Here are some obtuse angles.



How many things have you learned about lines and angles that you never thought about before?



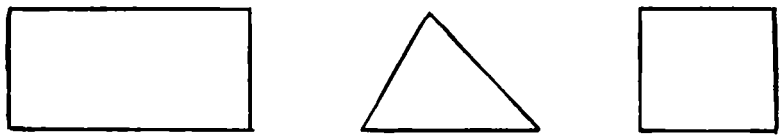
PLANE FIGURES AND POLYGONS

Where can you see a flat surface?

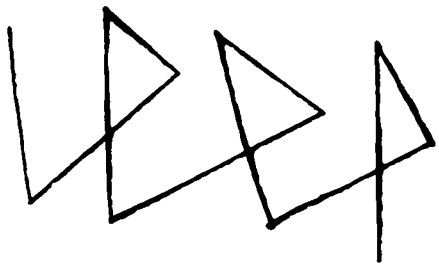
The walls of your room are flat surfaces. So is the floor and so is the ceiling. A table top is a flat surface, and a sheet of paper and a skating rink. In the physical world, a flat surface is an example of what mathematicians call a *plane*. The Latin word for "level ground" was *planus*, and the English word for a flat surface is *plane*.

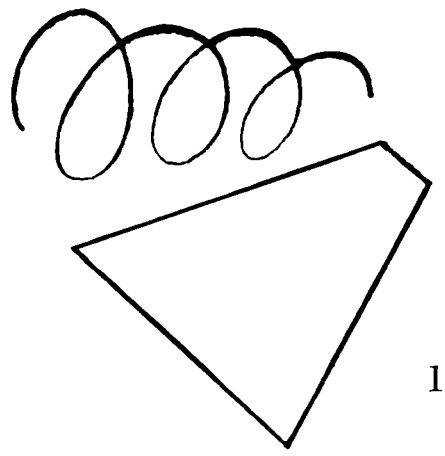
A plane, like a line, extends indefinitely, so you can't draw the idea of a plane any more than you can draw the idea of a line. But you can draw a line around part of a plane, and get a *plane figure*.

Here are some plane figures you've seen before.



These are plane figures too.

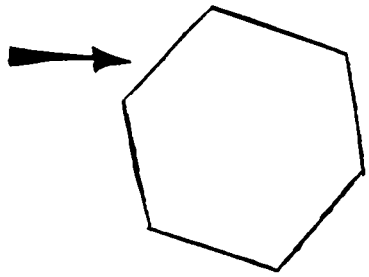


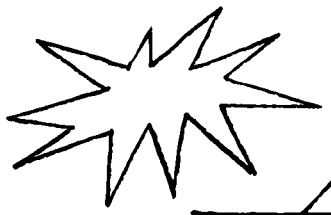


When you write a word, is that a plane figure?

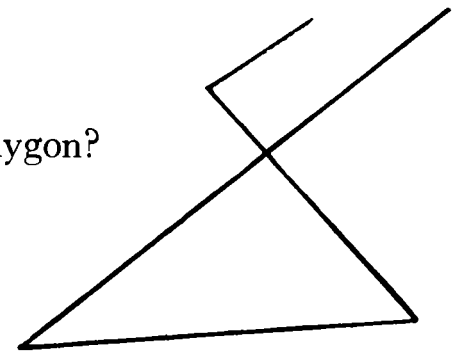
Many of the plane figures that we study in mathematics are called *polygons* (pol' i gons).

Polygon means "many-angled." A polygon has straight sides. It is closed, which means that the ends of the line segments always meet.

A polygon can look like this 

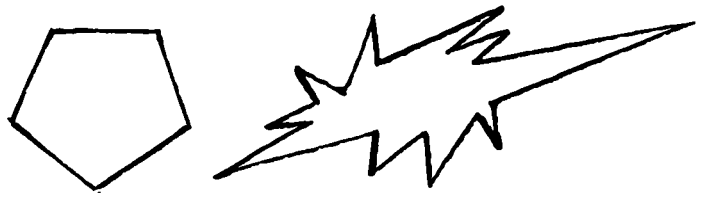
or this 

or this.

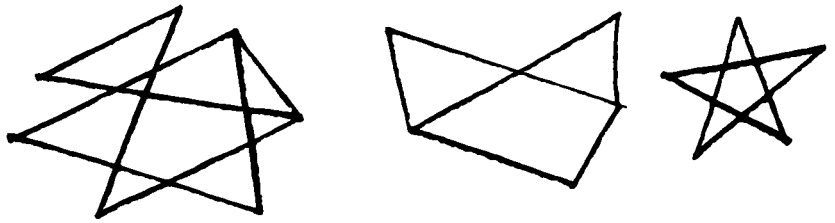
Is this a polygon? 

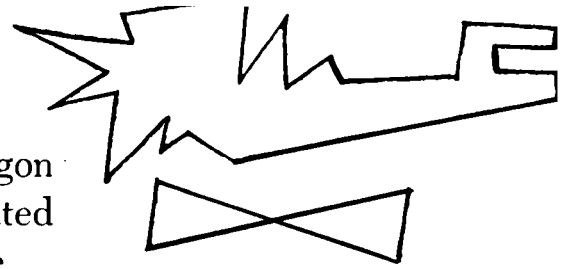
No, because the end points of the line segments do not meet. A polygon must be closed.

These are simple polygons. No sides meet, except at their end points.



Can you see why these are not simple polygons?



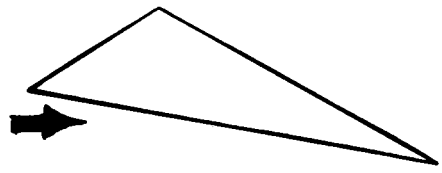


Sometimes a simple polygon looks much more complicated than one which isn't simple.

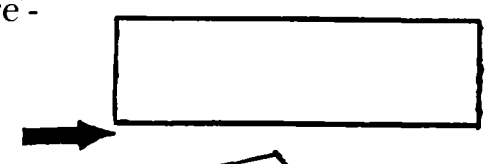
This seems confusing until you notice that a simple polygon has only one inside and one outside.

Polygons are named according to the number of sides they have. We still use the names the Romans gave them, thousands of years ago.

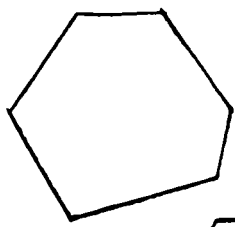
A triangle has three sides.
(*tri* means 3)



A quadrilateral (kwod re'-lat' er el) has four sides.
(*quad* means 4)



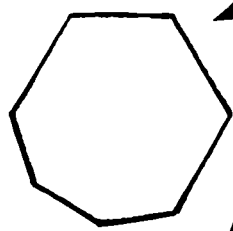
A pentagon has five sides.
(*pent* means 5)

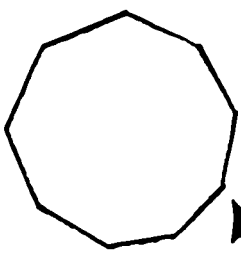


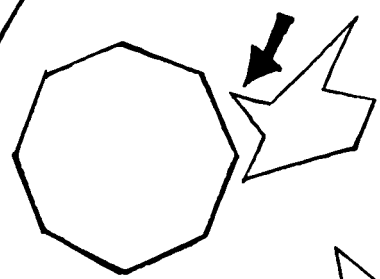
A hexagon has six sides.
(*hex* means 6)

A septagon has seven sides.
(*sept* means 7)

An octagon has eight sides.
(*oct* means 8)

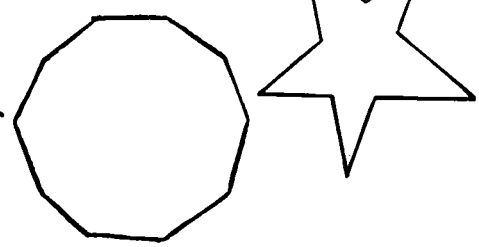




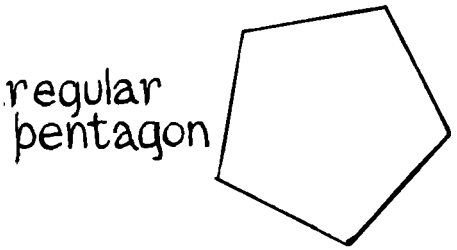


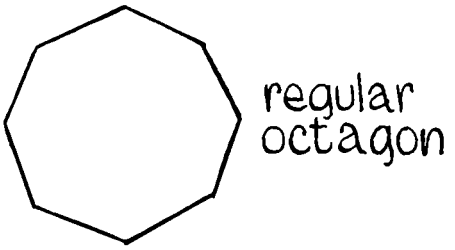
A nonagon has nine sides.
(*non* means 9)

A decagon has ten sides.
(*dec* means 10)

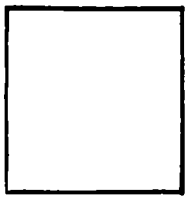


If all the sides of a polygon are the same length, and all its angles are equal, it is a *regular* polygon.





Do you know what a regular quadrilateral is called? It looks like this. A square, of course!

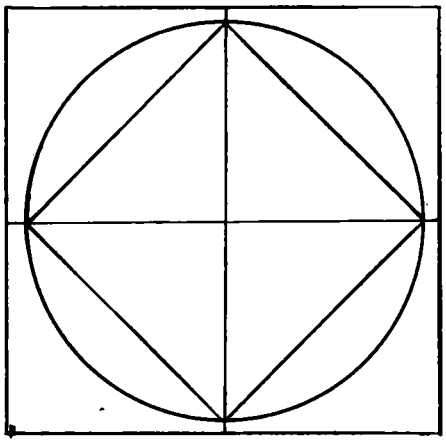


You can make some regular polygons without measuring.

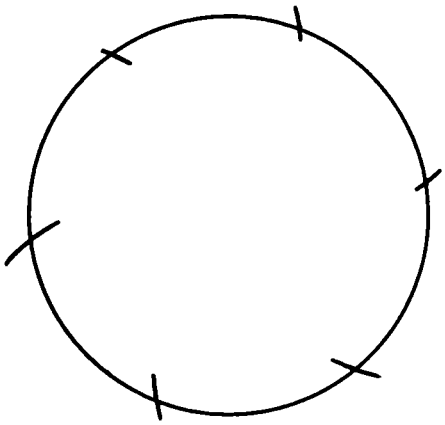
A square is easy. Remember how you folded the paper to get right angles?

Using the point of intersection as the center, draw a circle with your compass.

Draw the lines connecting the points where the folds cross the circle, and there's your square!

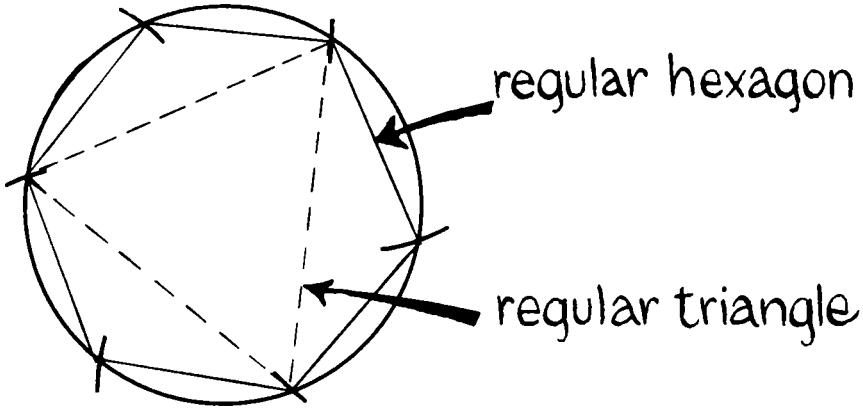


A hexagon is easy too. Draw a circle of any size with your compass. Be sure not to change the opening of your compass when you have finished. Now, starting at any place on the circle, mark off sections with your compass, like this



each time, put the compass point on the last mark you made

When you have gone completely around, you will have six places marked on your circle. Connect them with lines, like this



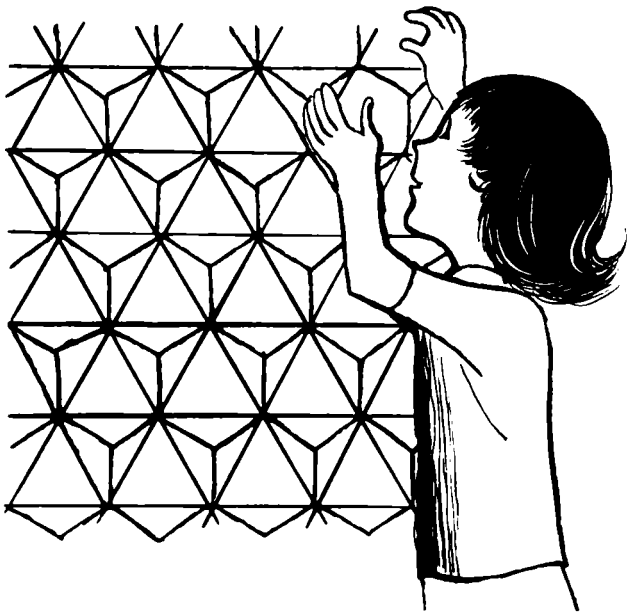
and there is your regular hexagon.

If you want to get a regular triangle, which is called an *equilateral* (e′ kwe lat′ er el) triangle, connect only every other mark.

We said that a line segment has one dimension, length.

How many dimensions has this polygon?

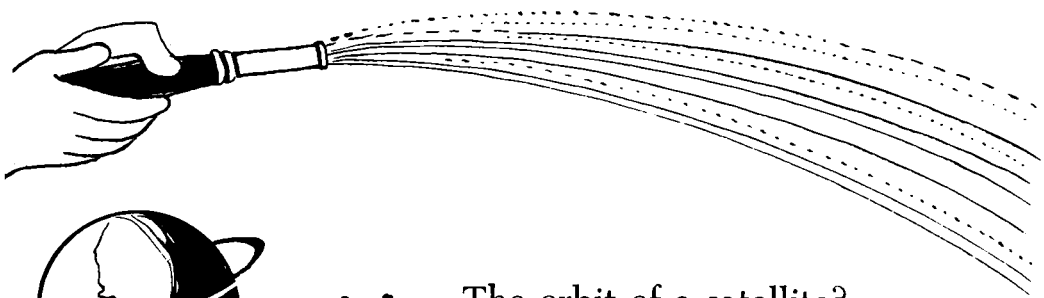
Two, length and width.

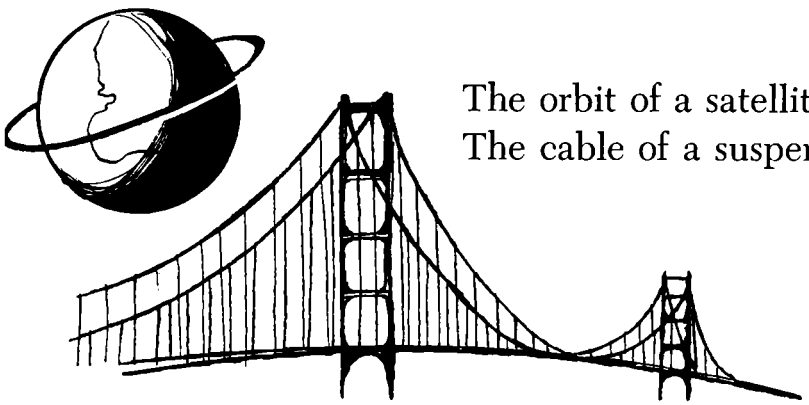


you can make beautiful designs with regular polygons

MATHEMATICS AND CURVES

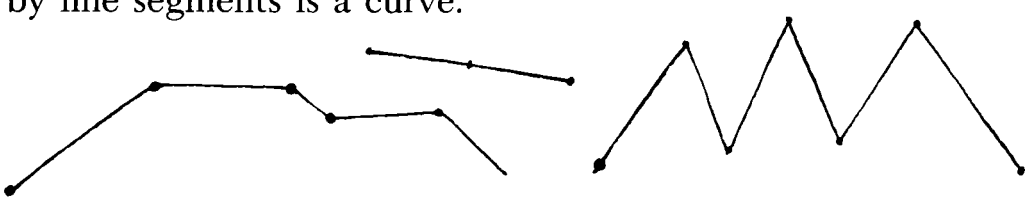
When we talk about curves, what picture comes into your mind? Do you think about a pitcher throwing a baseball? Water coming out of a hose?





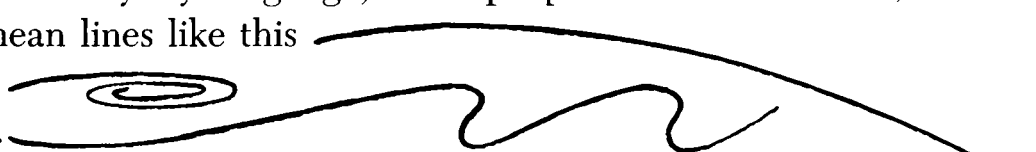
The orbit of a satellite?
The cable of a suspension bridge?

Some mathematicians say that any set of points connected by line segments is a curve.



Both of these figures would be called curves. Some mathematicians say that even a straight line is a curve!

But in everyday language, when people talk about curves, they mean lines like this

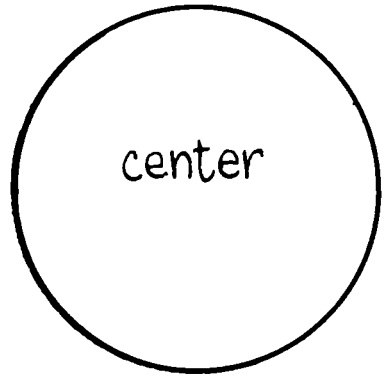


or this

or this.

Some curves extend on and on,
and some are closed curves.

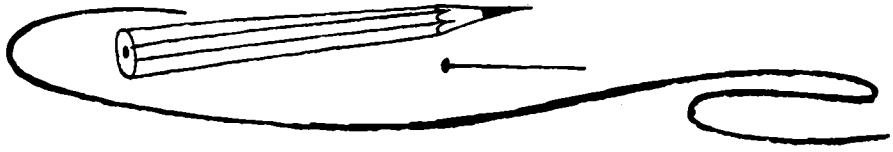
CIRCLES



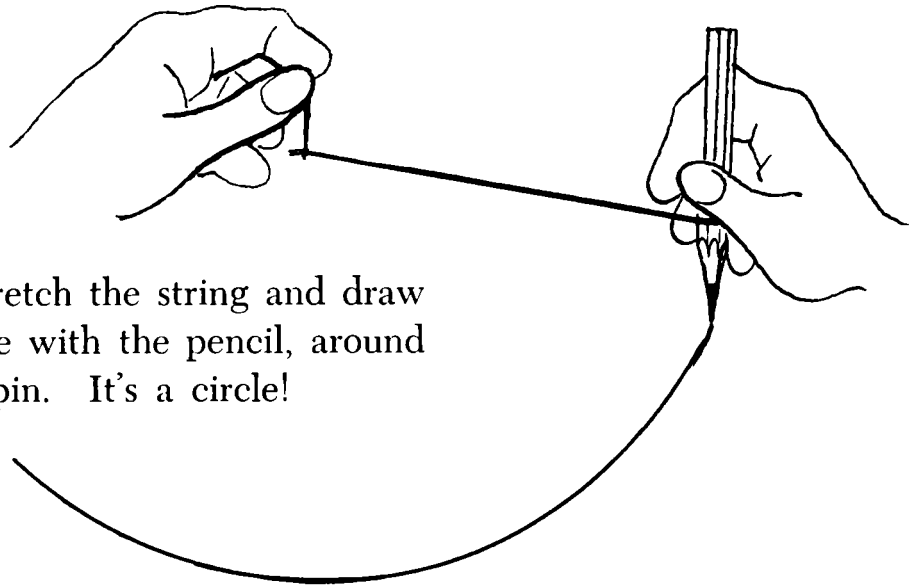
A circle is a closed curve.

It is a plane figure. Every point on the curve is the same distance from a point which we call the center. A circle is an easy figure to draw, even without a compass.

You need a pencil, paper, a piece of string and a pin.



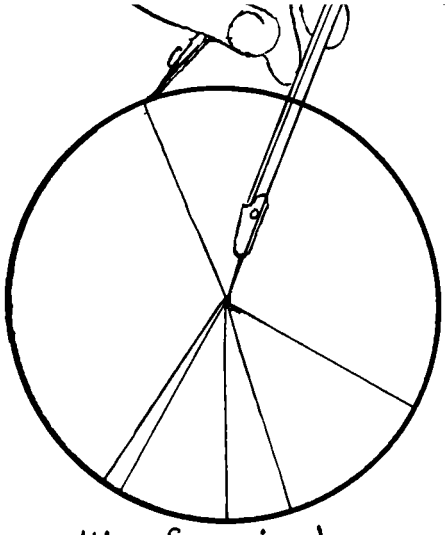
Tie one end of the string around the pencil and put the pin through the other end. Stick the pin in a piece of paper and hold it upright with your finger.



Stretch the string and draw a line with the pencil, around the pin. It's a circle!

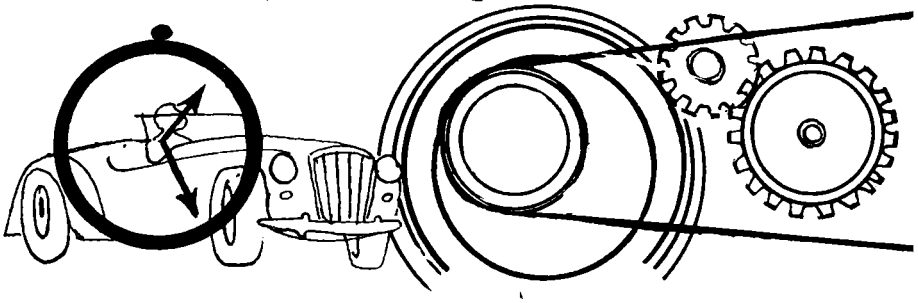
The distance between the center of the circle, where the pin is, and any point on the circle is always the same. You can see that it would have to be, because the pencil was always the same distance from the pin.

A line segment joining the center to any point on the circle is called a *radius.* The string, in any position, was the radius of the circle you drew. (More than one are called *radii.*) Did the string ever get onger or shorter? No. All radii of a circle are the same ength.



all radii of a circle are the same length

There are no true circles in nature. The true circle is an invention of man, and one of his greatest ideas. How many things do you see around you that depend on man-made circles?

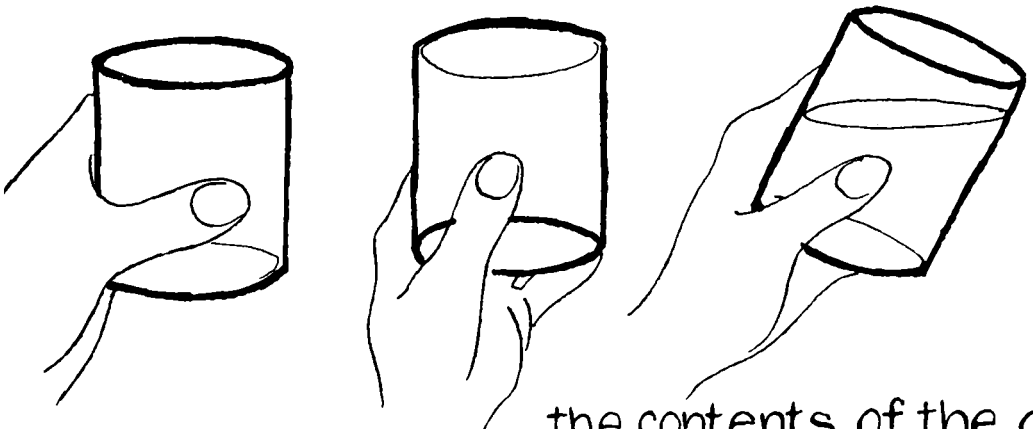


ELLIPSES

The top of a drinking glass is usually a circle, isn't it? But if you want to see it as a circle, you have to look straight down on it.

What happens to the circle if you hold the glass in front of you, above or below the level of your eyes?

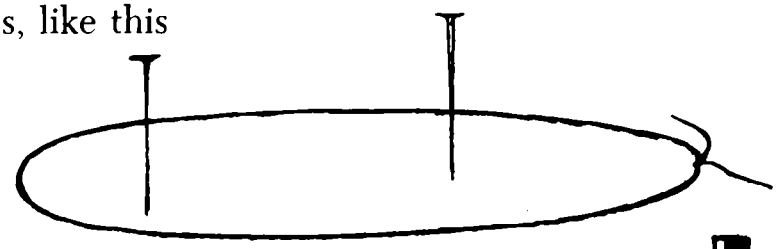
The shape looks different. Instead of seeing a circle, you see an *ellipse.*



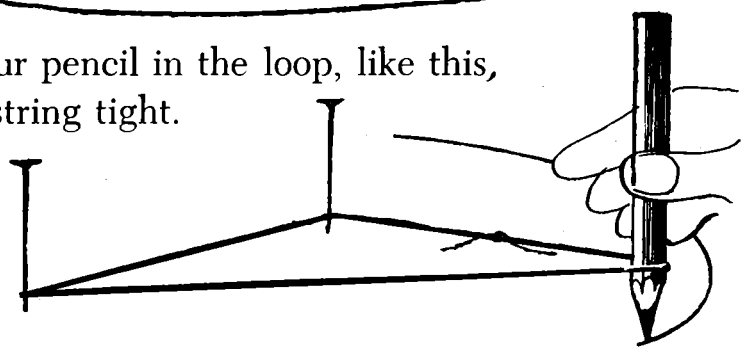
the contents of the glass form an ellipse too

Drawing an ellipse with a pencil, string and pins is a good way to find the difference between an ellipse and a circle. To draw an ellipse you need *two* pins.

Put the pins in a piece of paper about two inches apart. Tie an eight inch string into a loop, and place the loop over the pins, like this



Now put your pencil in the loop, like this, pulling the string tight.



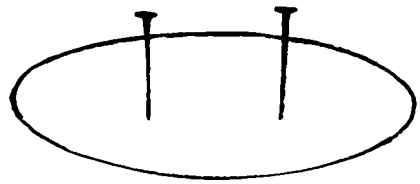
Keep the string tight and draw a line completely around the pins.

You have made an ellipse.

What is the difference between a circle and an ellipse?

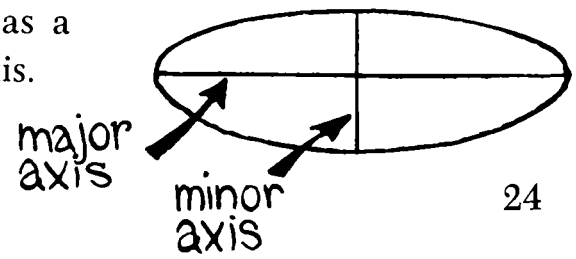
An ellipse has two centers, or *foci*, while a circle has only one center, or focus.

each pin is a focus

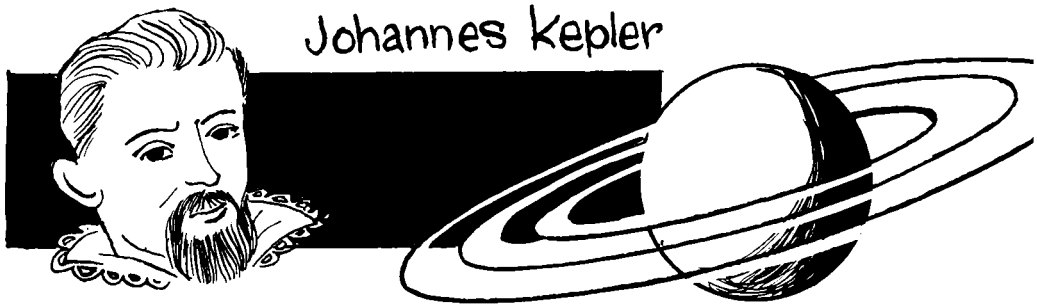


Remember we said that every radius in one circle is the same length? Look at your ellipse. Can you see why that couldn't be true of an ellipse?

Instead, every ellipse has a major axis and a minor axis.



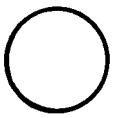
The ellipse is a very important curve in astronomy. Many many years ago, when scientists first discovered that the earth and the other planets moved around the sun, they thought that they moved in circular paths. When, later, a famous astronomer named Kepler did some very complicated mathematics he discovered that their paths, or *orbits*, were not circles but ellipses.



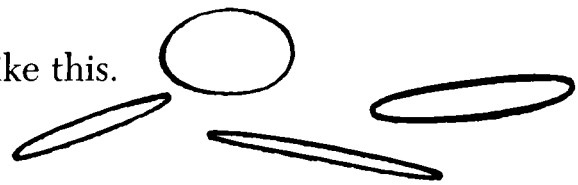
Then where are the two foci that such an ellipse must have?

The center of the earth is one of the focal points. The other is out in space.

Circles may be different sizes, but they all have the same shape. They are all round.

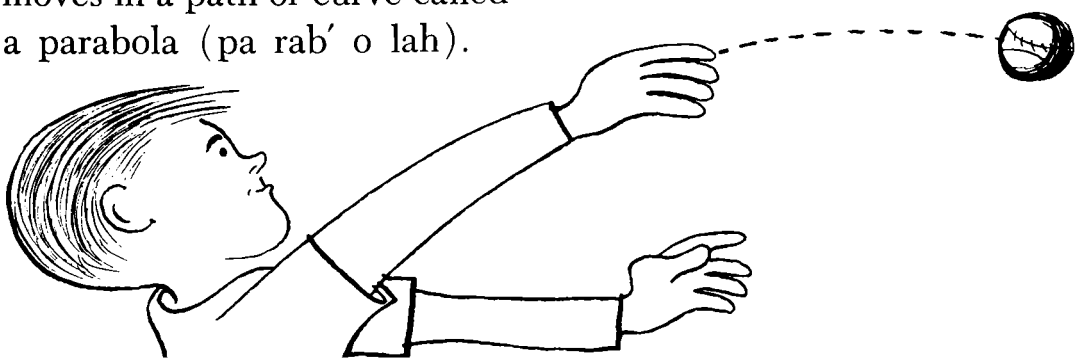


But ellipses are not all the same shape. Some can be almost round,
and some can look like this.



PARABOLAS

Anything thrown into the air—
like a baseball,
or water from a hose,
or a bullet shot from a gun,
moves in a path or curve called
a parabola (pa rab′ o lah).



The object moves in a curve because the moment it is free, the earth's gravity starts pulling it down.

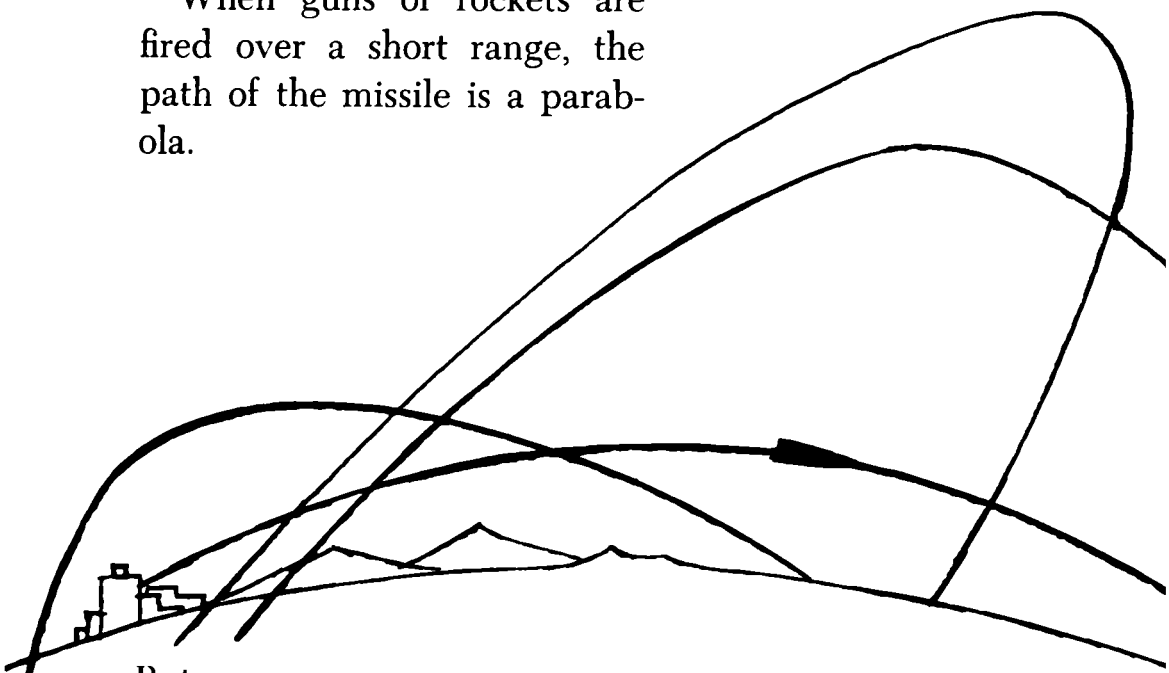
A parabola looks almost like part of an ellipse, but there is a big difference between the two curves.

An ellipse is a closed curve.

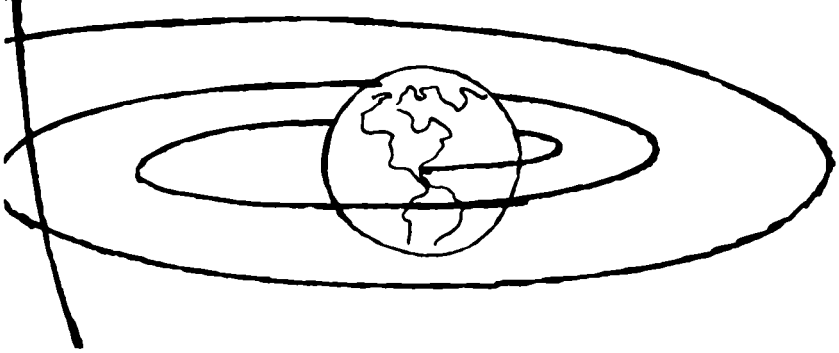
A parabola is a curve that is not closed. It would continue widening forever, to infinity.

This idea is very important to rocket scientists.

When guns or rockets are fired over a short range, the path of the missile is a parabola.



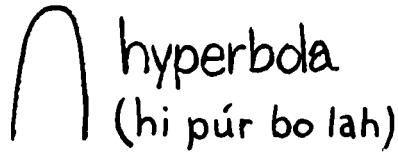
But once things are fired into space, their paths are not parabolas, but ellipses.



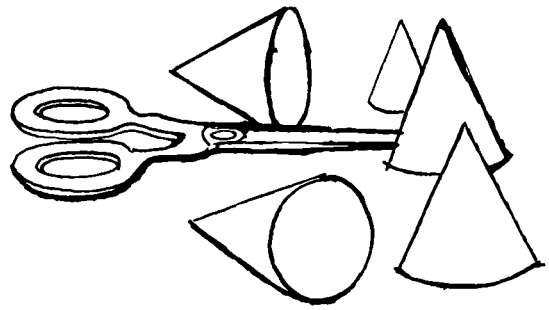
Isn't it interesting that moving bodies which go a short distance and are attracted only by the earth's gravity take paths that are parabolas? But most moving bodies in space move in ellipses.

HYPERBOLAS

There is another kind of curve that looks a great deal like a parabola. It is called a *hyperbola.*

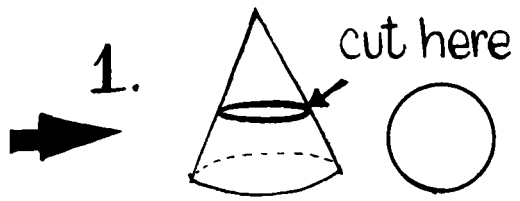


You can learn a lot about all these curves, and what makes them different, by making them yourself out of paper cups. You'll need some cone-shaped paper cups and a straight scissors.

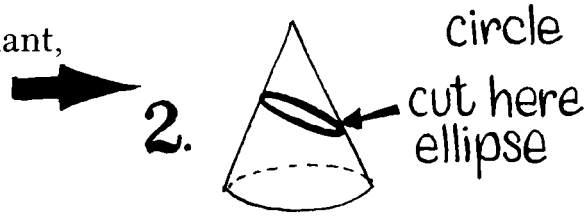


Stand the cups upside down on a flat surface.

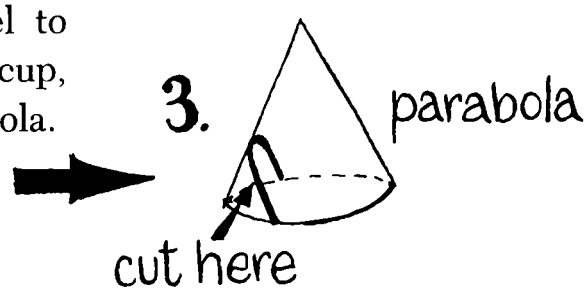
Cut one cup through the middle, parallel to the base. You will see that you have cut a circle.

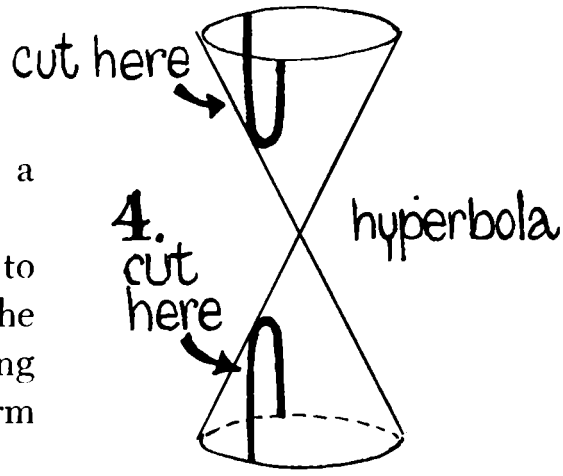


Cut the second cup on a slant, like this. Isn't it an ellipse?



Cut the third cup parallel to the opposite side of the cup, and you will have a parabola.



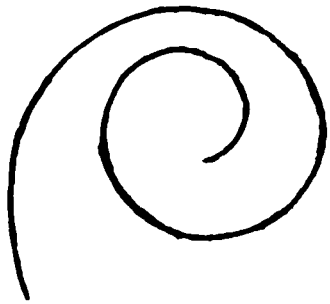


You need two cups to make a hyperbola.
Cut each cup at right angles to its base. If you place the cones tip to tip, the facing curves you have cut out form a hyperbola.

Can you guess why these curves are called *conic sections*? The Greeks were experts on conic sections almost 2,500 years ago.

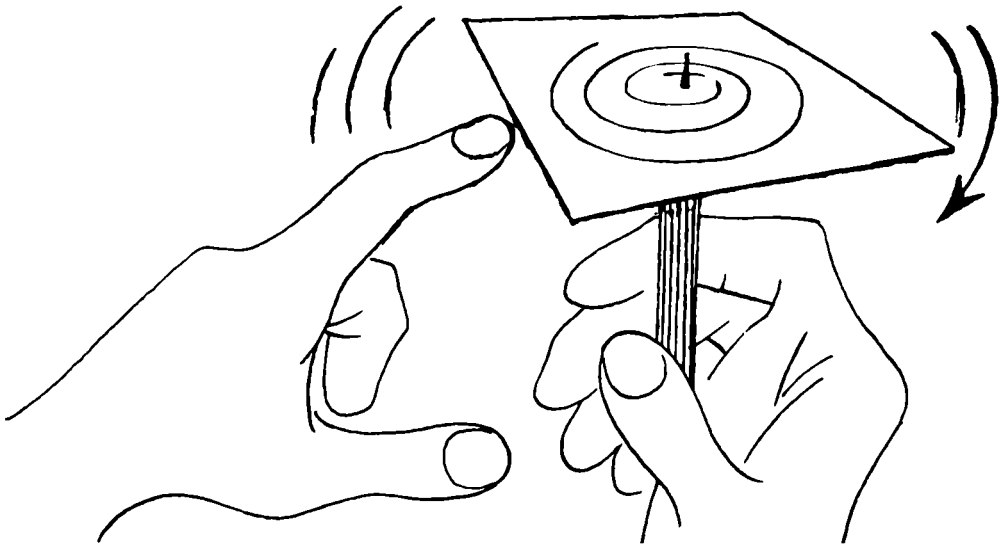
OTHER CURVES

What do you know about the curve we call a *spiral*? Try this.



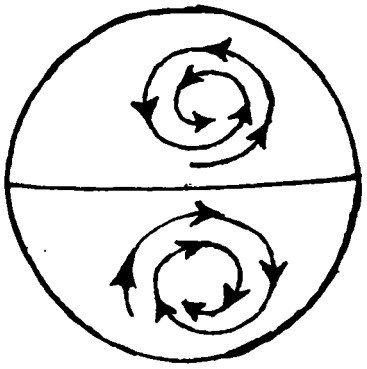
Draw a spiral on a piece of cardboard.

Put the point of the pencil through the center of the spiral and twirl the cardboard. When you twirl it one way, the curve seems to move outward. When you twirl it the other way, the curve seems to move toward the center.



Have you ever noticed, when you take the stopper out of the sink, that the water empties in a spiral pattern? If you live above the equator, the water moves counterclockwise. If you live below the equator, the spiral moves clockwise.

Spiral-shaped storms, like hurricanes or cyclones, do the same thing.



One of the most useful curves is a helix.

A helix is like a spiral, but instead of being a plane figure it looks like this

Can you see what's different about it?

If you have ever unwrapped the paper wound around a colored pencil, you have seen a helix. The threads on a screw are really a helix. You can see these helix screws on electric light bulbs too, and on drills, pencil sharpeners and automatic pencils.

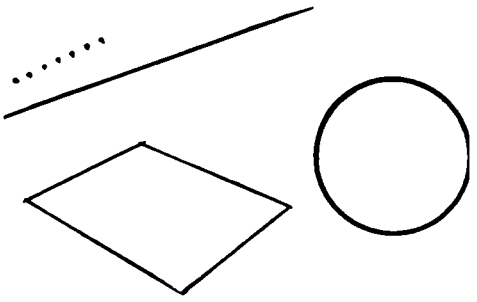
What makes a helix different from a spiral?

It has a *third dimension.*

SOLID FIGURES

We have talked about points, which have no dimension.

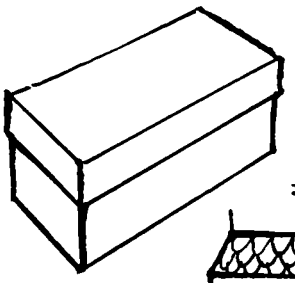
Then we talked about sets of points which gave us lines, and line segments, which have one dimension,
and other sets of points which gave us planes,
and plane figures, which have two dimensions.



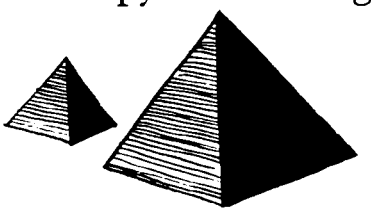
Are there any figures which are not lines or planes?

Certainly.

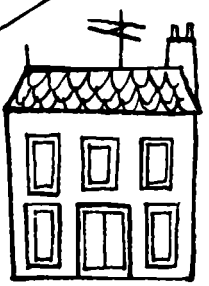
Think about a shoebox,

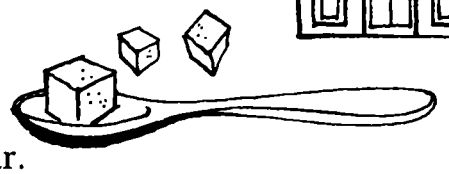


or the pyramids of Egypt,



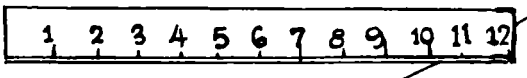
or a house,



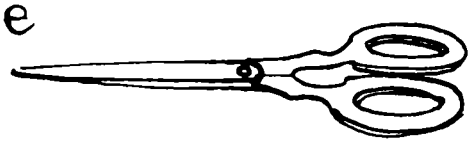


or a cube of sugar.

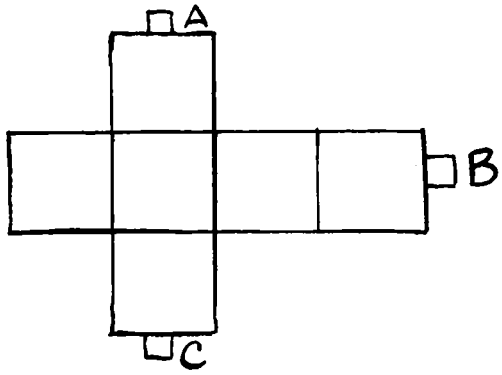
These are not flat. They are called *solid figures.*
Let's make the solid figure called a cube.

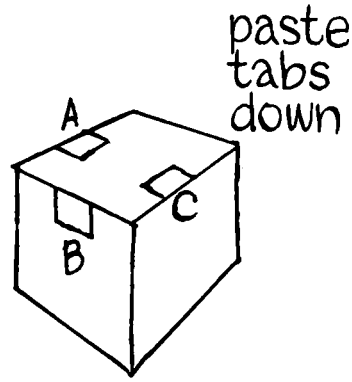


You need a piece of paper, a ruler, scissors and paste



Cut out a piece of paper four squares wide, with another square on each side of the second square, like this

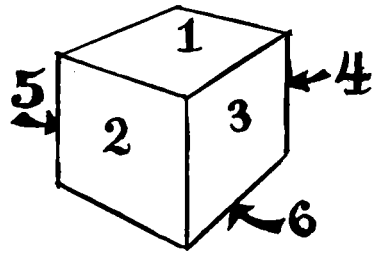




Leave tabs, as shown, for pasting.
Fold your squares up to form a cube, like this.

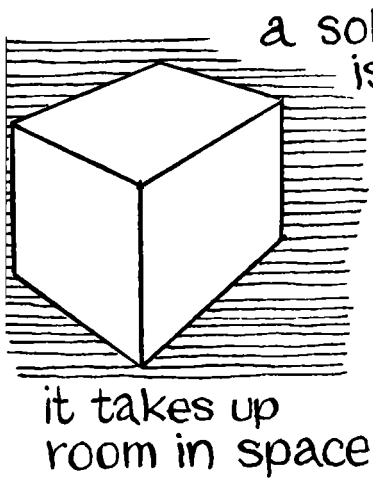
How many faces does the cube have? Six.

It also has an outside and an inside. All solid figures do.



But the inside of this cube is hollow! Then why do we say it is a solid?

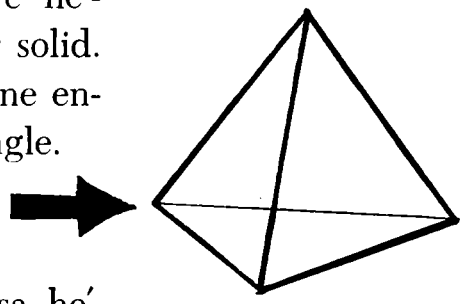
It really doesn't matter, in mathematics, whether the inside of a figure is filled or empty. If it is a closed surface, taking up room in space, enclosing a portion of space, it is called a solid figure.



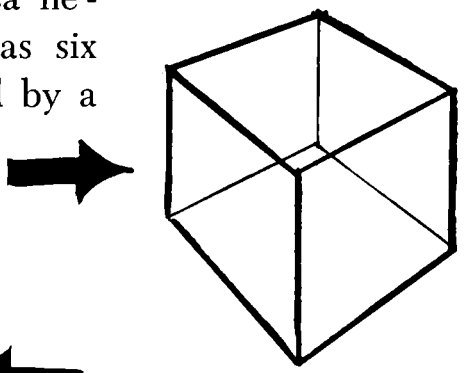
The solids we have been talking about are called *polyhedrons.* (pol e he′ drons) *Hedra* is the Greek word for "base" or "seat," so a polyhedron has many bases.

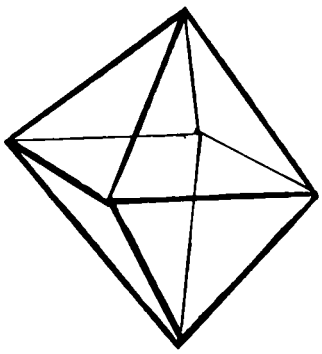
There are only five regular solids in the world. A regular polyhedron is one in which all the bases, or faces, are the same size and shape, and all its angles are equal.

A tetrahedron (tet′ re he′-dron) is the first regular solid. It has four faces, each one enclosed by a regular triangle.



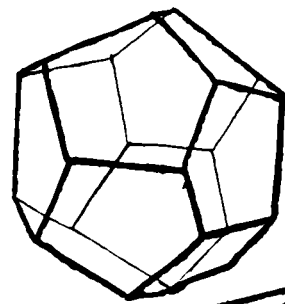
A hexahedron (hek′ sa he′-dron) is a cube. It has six faces, each one enclosed by a square.



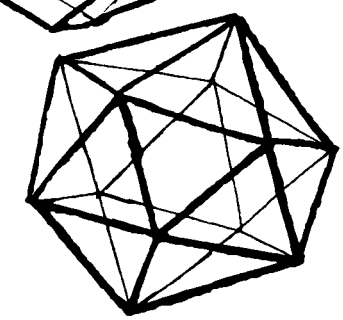


An octahedron (ok′ ta he′-dron) is the third regular solid. Its eight faces are all enclosed by regular triangles.

A dodecahedron (do′ dek a-he′ dron) has twelve faces, each enclosed by a regular pentagon.

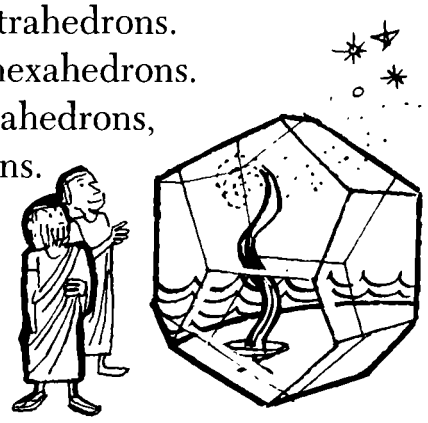


An icosahedron (i′ ko sa he′-dron) is the fifth regular solid. Each of its twenty faces is enclosed by a regular triangle.



The ancient Greeks thought that the world was made up of four elements: fire, earth, air and water. They had discovered the regular solids, and they thought that each element was made up of one of them.

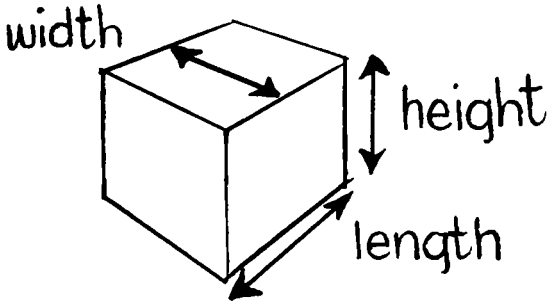
They thought fire was made of tetrahedrons.
They thought earth was made of hexahedrons.
They thought air was made of octahedrons,
and water was made of icosahedrons.



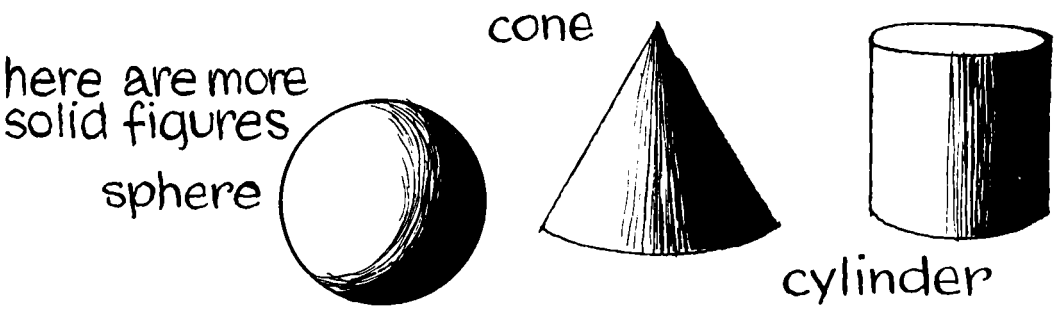
They put them all together inside a dodecahedron and said that the dodecahedron was the symbol of the whole universe.

Today we know that there are more than a hundred elements, and scientists keep discovering new ones. But the dodecahedron is still used as the symbol of the universe.

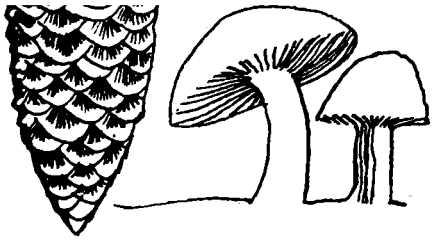
A solid, such as a cube, is said to have three dimensions, length, width and height.



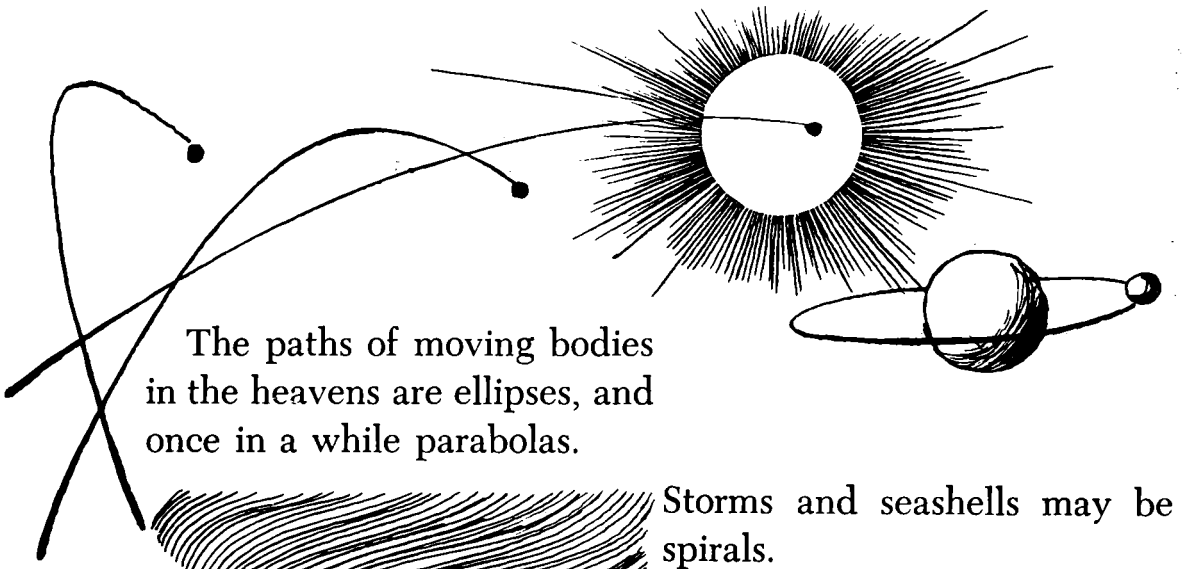
Of course all the figures we have made here are really plane figures, drawn to *represent* solid figures. You can't make a solid figure, which has three dimensions, on a page which has only two dimensions!



SHAPES IN NATURE



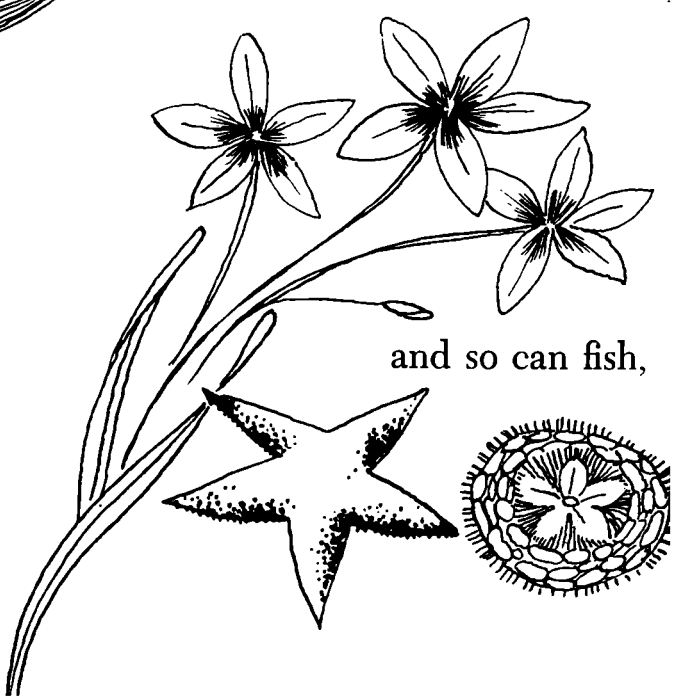
Every shape that a mathematician or an artist ever dreamed of, existed in nature long before man thought of mathematics. Any day, just by looking, you can see straight lines and curves, cones and triangles, spheres and polygons.



The paths of moving bodies in the heavens are ellipses, and once in a while parabolas.

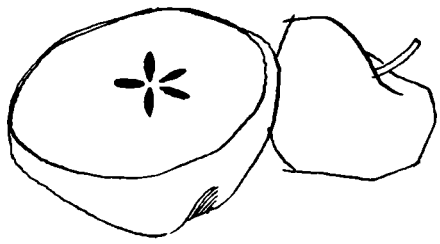
Storms and seashells may be spirals.

Flowers can be star-shaped

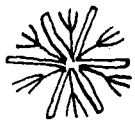


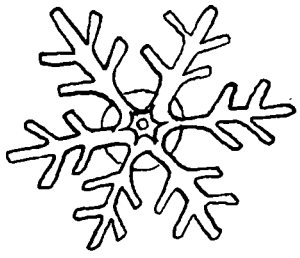
and so can fish,

and if you cut an apple crosswise you will see a five-pointed seed star.

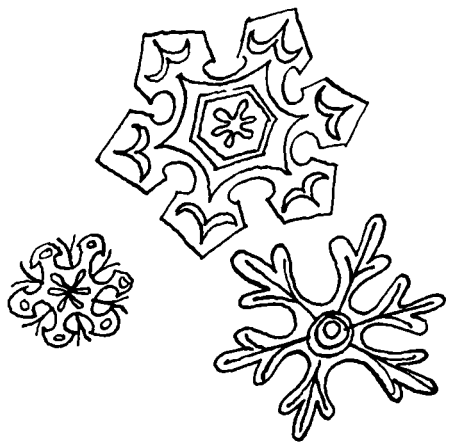


Under a microscope you can see

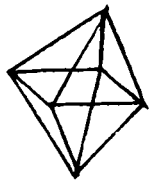




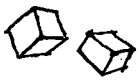
that snowflakes are hexagons,



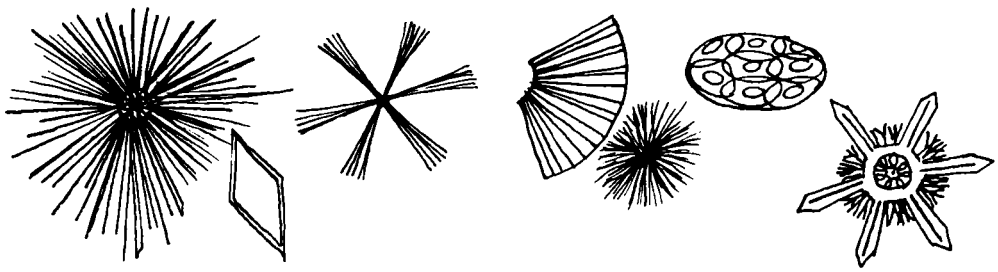
diamond crystals are octahedrons,



salt crystals are cubes.



And at the bottom of the ocean scientists have found billions of skeletons of microscopic plants and animals, as old as life and as beautiful, all with mathematical shapes.



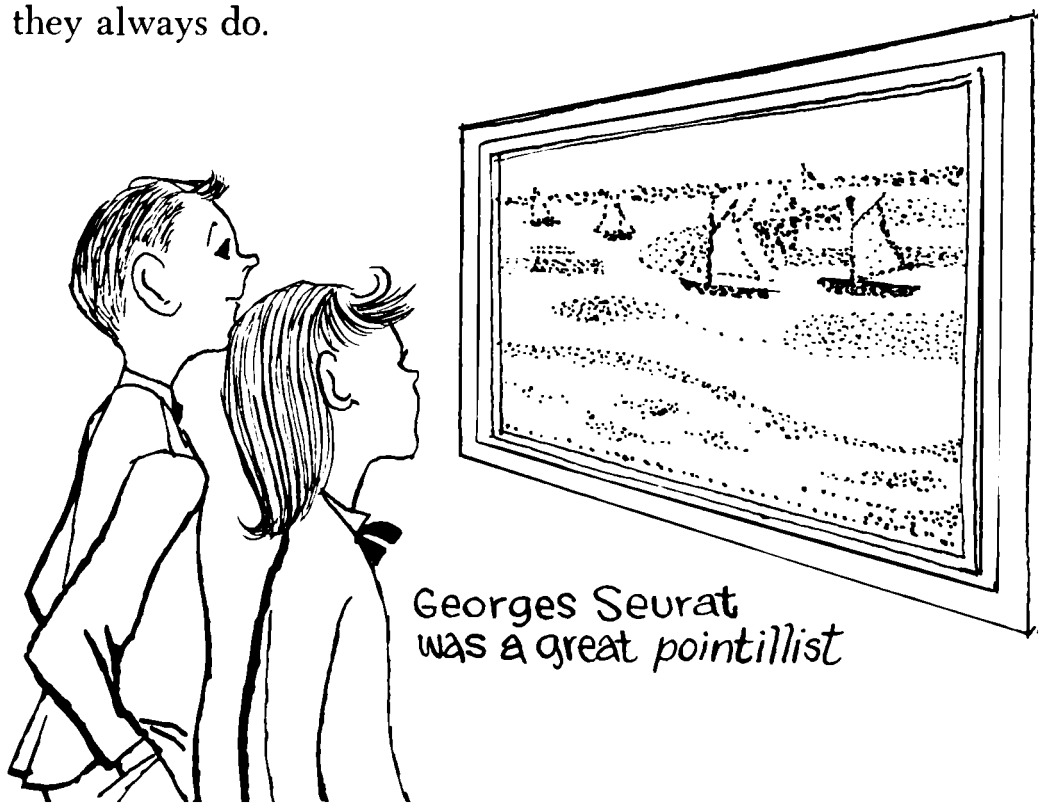
MATHEMATICS IN ART

What has mathematics to do with art?

Everything we've talked about so far—points, lines, curves and angles, planes and shapes are all a part of art. We can't really understand art unless we know about them.

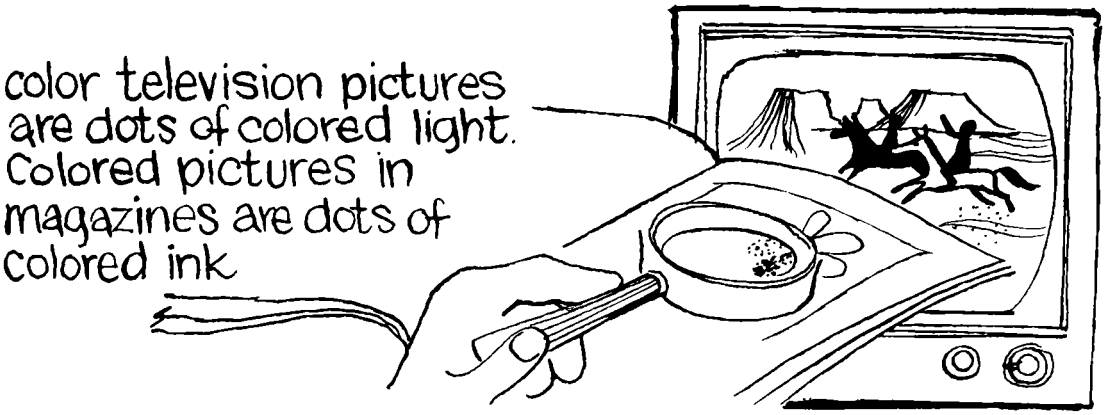
But how can a point, or even the dot which marks the point, be used by an artist?

We know that all light can be broken up into dots of different colors. About a hundred years ago a group of French painters had the idea that they could do the same thing with paints. They decided to paint with dots, or points of color. If you stand close to their paintings, you see only thousands of tiny dabs of color. But if you stand away, the dabs become a picture, each object shimmering with light. The artist did not need to blend the colors. Your eyes do it, as they always do.

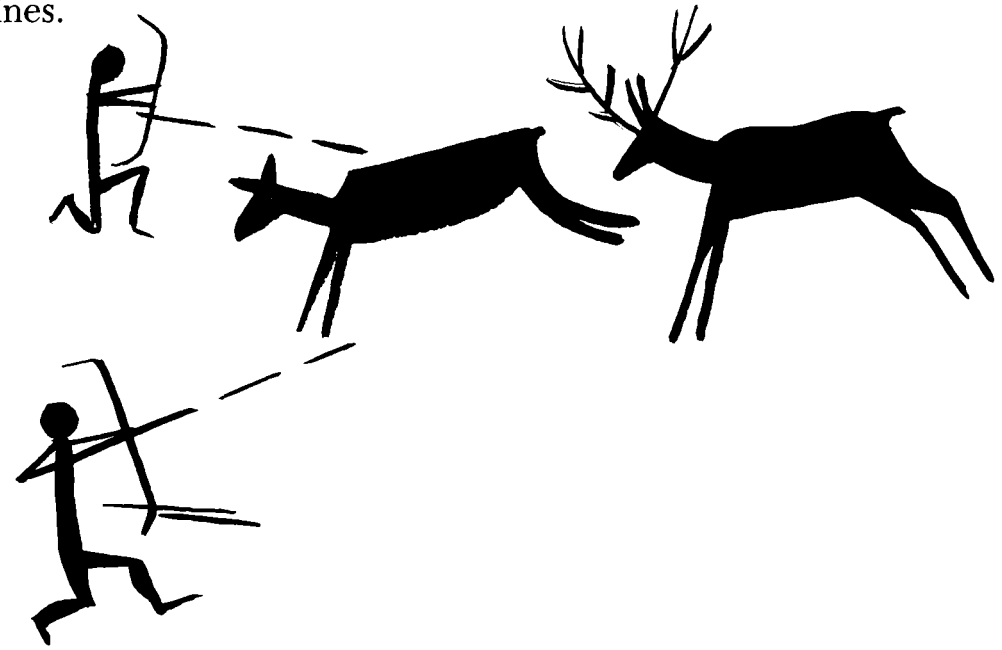


Do these artists sound like kindergarten painters? Their paintings hang in the great art museums of the world. The artists are called the *pointillists*—the point painters.

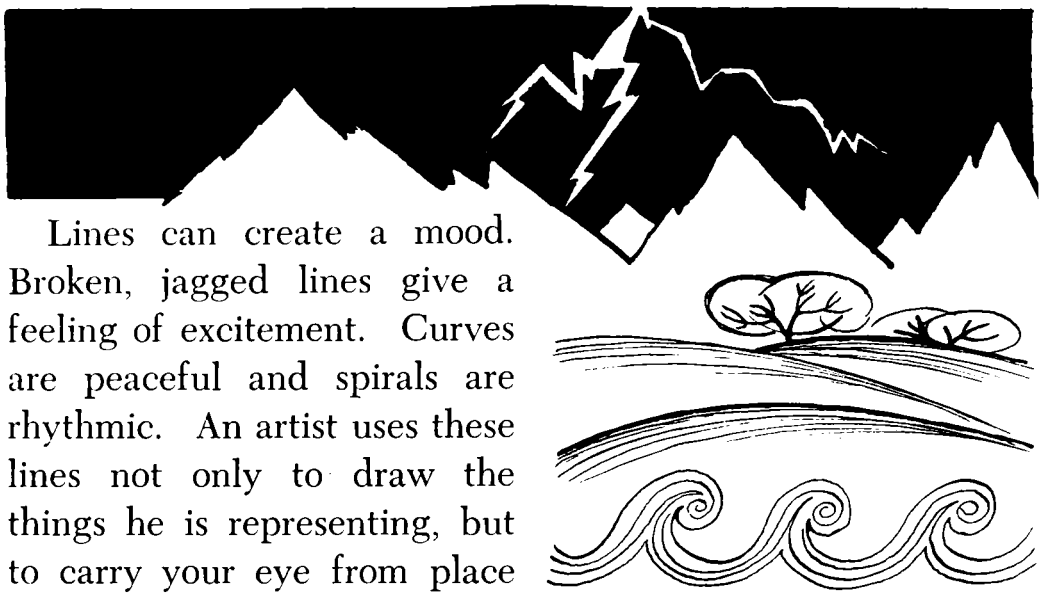
Is it hard to believe you can make a picture out of dots of light, or dots of color? The colored pictures in your magazines are made of dots of color—you can see them if you look carefully (a magnifying glass will help you to see them better). Color television pictures are dots of colored light.



You can't draw without lines. Even the cave men discovered that they could represent the things around them with lines.

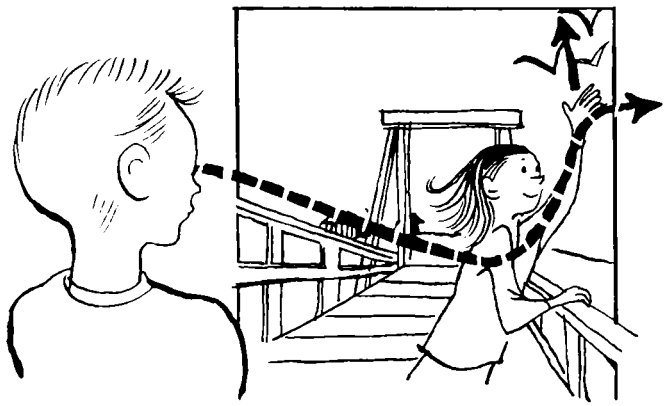


You can recognize the work of many artists by the special kinds of lines they make, just as you can recognize someone's handwriting.

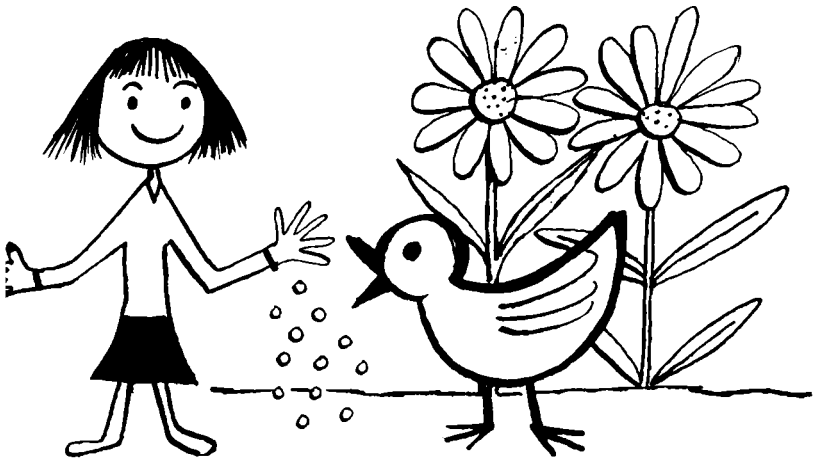


Lines can create a mood. Broken, jagged lines give a feeling of excitement. Curves are peaceful and spirals are rhythmic. An artist uses these lines not only to draw the things he is representing, but to carry your eye from place to place in the picture.

If you look carefully at a painting or a billboard, a magazine page or a book illustration, usually you can find that the artist has given your eyes an entrance, has led you through the picture, and has pointed the way out.



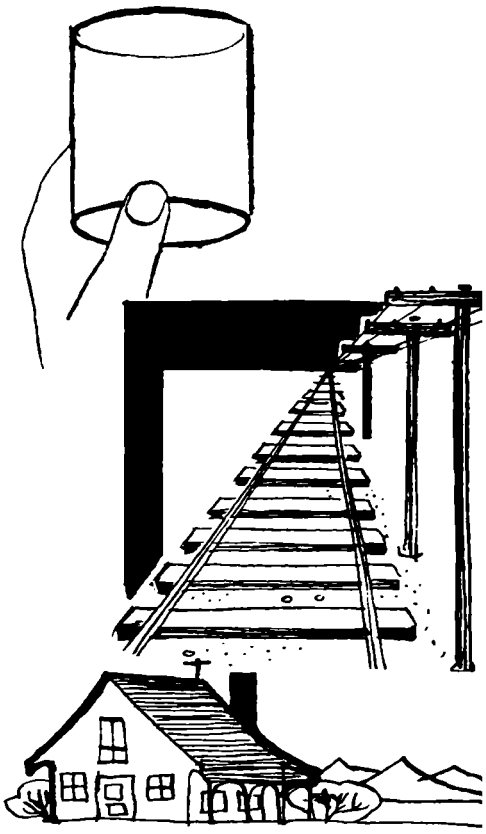
The page of a book, a sheet of drawing paper, a canvas or a wall are all flat surfaces. An artist has to *draw* the third dimension into a two-dimensional plane. It took a long time for artists to learn how to do this.



Remember how you used to draw when you were little? You made the most important thing in the picture the biggest, whether it was really large or small, near or far. Early artists did the same thing. They did not know how to put things in the background. Have you ever noticed, in ancient Greek or Egyptian paintings, that everyone seems to be standing in a line?

The ancient Chinese painters knew how to make some things look closer than others, but they did it with color. They painted the things they wanted to appear closest in deep colors, and the faraway things in pale colors.

About 500 years ago certain mathematical rules were discovered that made it possible for paintings to look as if they had a third dimension.



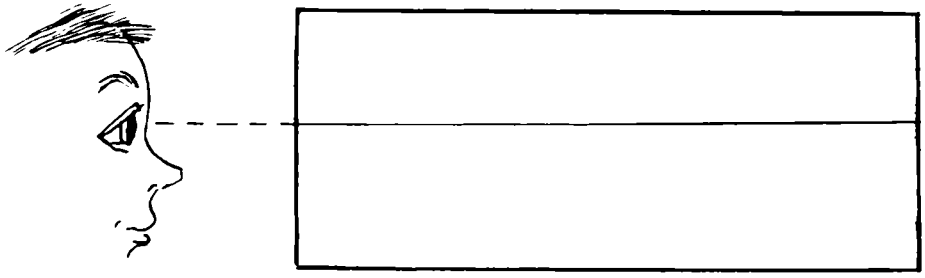
Remember the glass that was a circle when you looked directly into it? When you held it above or below eye level, your eyes saw the circle as an ellipse, even though your brain *knew* that the glass was a circle.

We *know* that railroad tracks are always parallel, yet they *look* as if they come together in the distance.

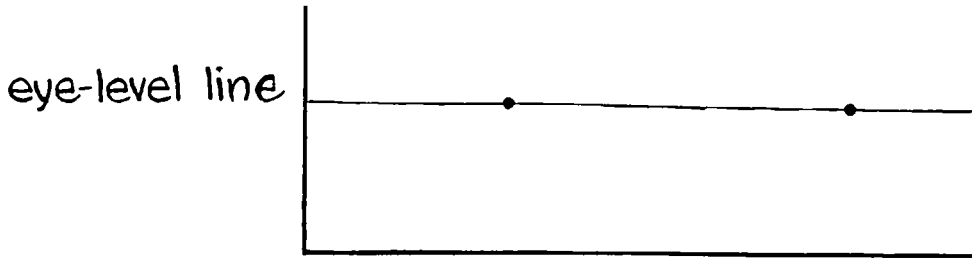
We *know* that a mountain is bigger than a house, but it *looks* smaller if the house is close to us.

In the fifteenth century, several Italian architects and artists became interested in *perspective*, which means "seeing through." One painter, Paolo Uccello, became so interested in constructing the mathematical rules of perspective that he almost stopped painting. He realized that artists had been looking at things with their brains instead of their eyes. He perfected a way of drawing, mathematically, what his eyes saw instead of what his brain knew.

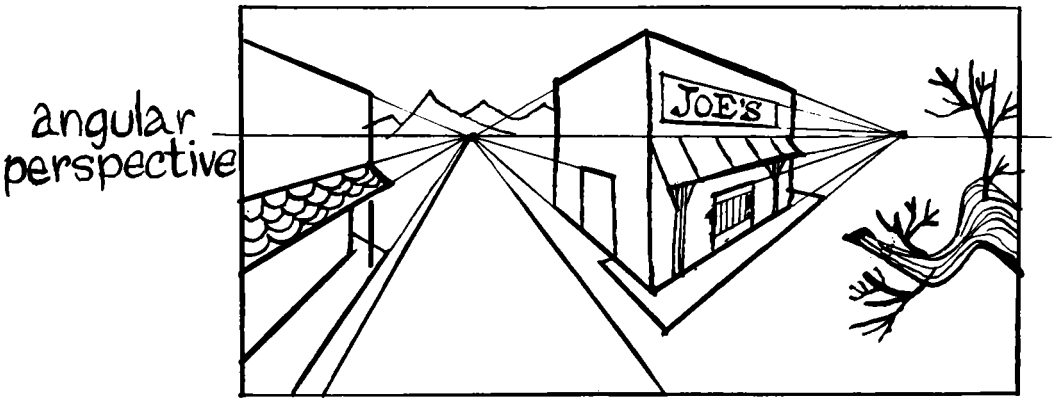
Do you know how to draw in perspective?



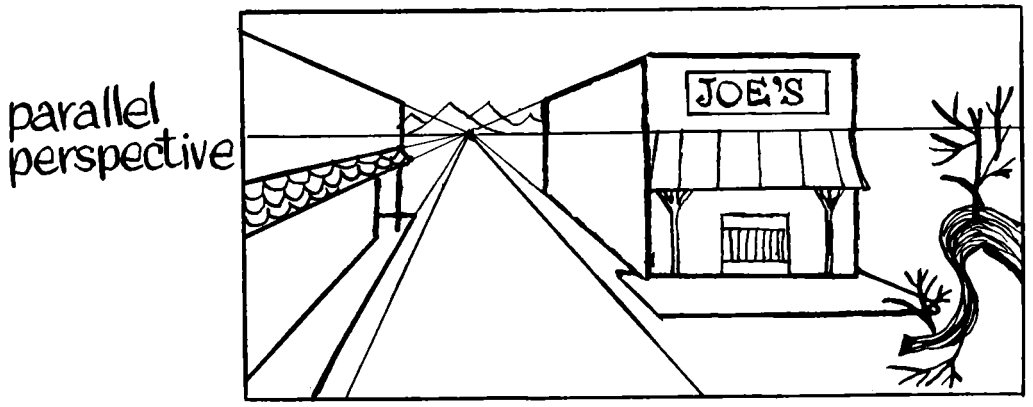
First draw a line where you want eye level to be in your picture.



Then, on the line, you can put one *vanishing point*
or two
or even more.
The lines of everything above your eye level line are drawn *down* to the vanishing points.



The lines of everything below eye level are drawn *up* to the vanishing points.
If you have only one vanishing point, the same picture looks like this.

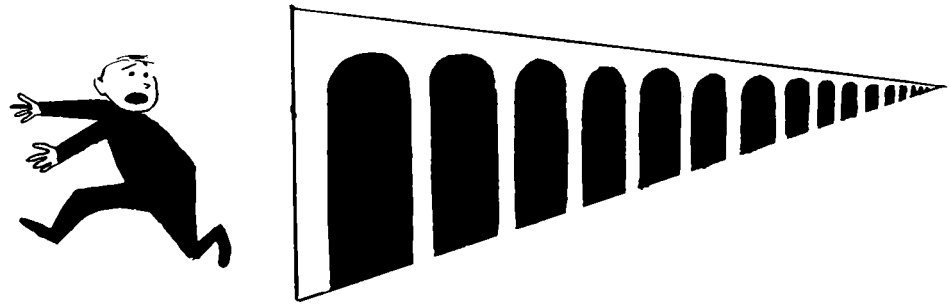


What do you notice about the other lines?
They are parallel.

This kind of perspective, with one vanishing point and the other lines parallel, is called parallel perspective.

Once artists knew the rules, and found that they worked, everyone became interested in putting a third dimension into painting. Just as a cube is more interesting than a square, a painting you can look into is more interesting than a flat one.

Perspective became almost a game. Artists delighted in painting long rows of columns and arches which led the eye farther and farther back into the canvas. But there was no way out, and the viewer became lost in the painting!

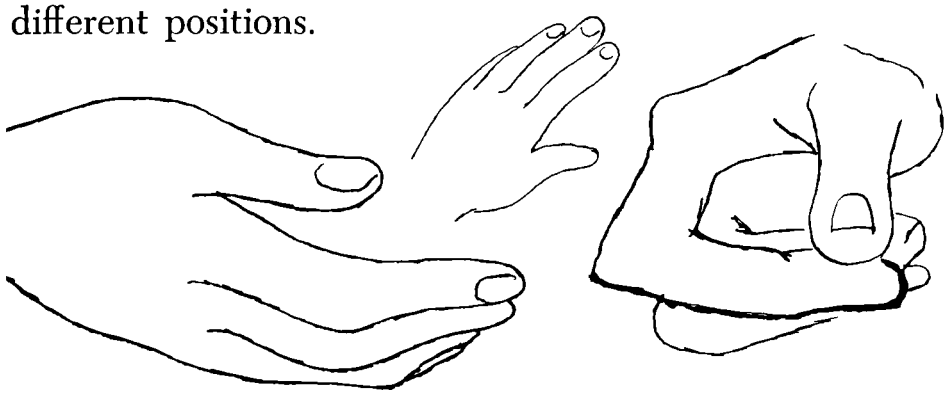


Later, artists began to look at things as though they had several pairs of eyes or as though they were moving from place to place. Then they learned to use two point perspective, as you did in the first drawing, and even perspective with more than two vanishing points. Can you see why perspective with more than one vanishing point is called angular perspective?

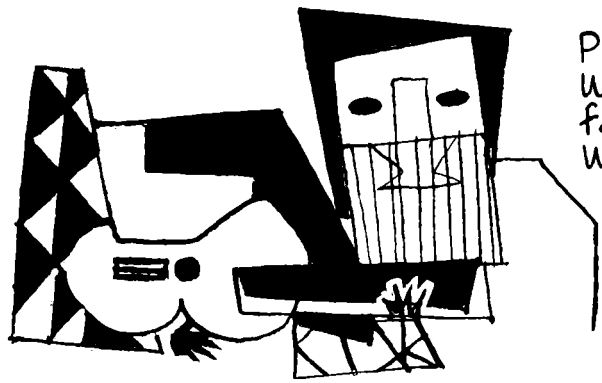
Artists learned to look at lines differently to get perspective, and they learned to look at shapes differently too.

Everything you see has a shape. You can draw these shapes with circles and squares, triangles, cubes, octahedrons or any other figures. But just as the shape of a glass changes when your eyes see it from a different place, so does every other shape.

The laws of perspective help here too. Artists use *foreshortening* to draw things from different viewpoints. Look at the way the shape of a hand changes when you see it from different positions.

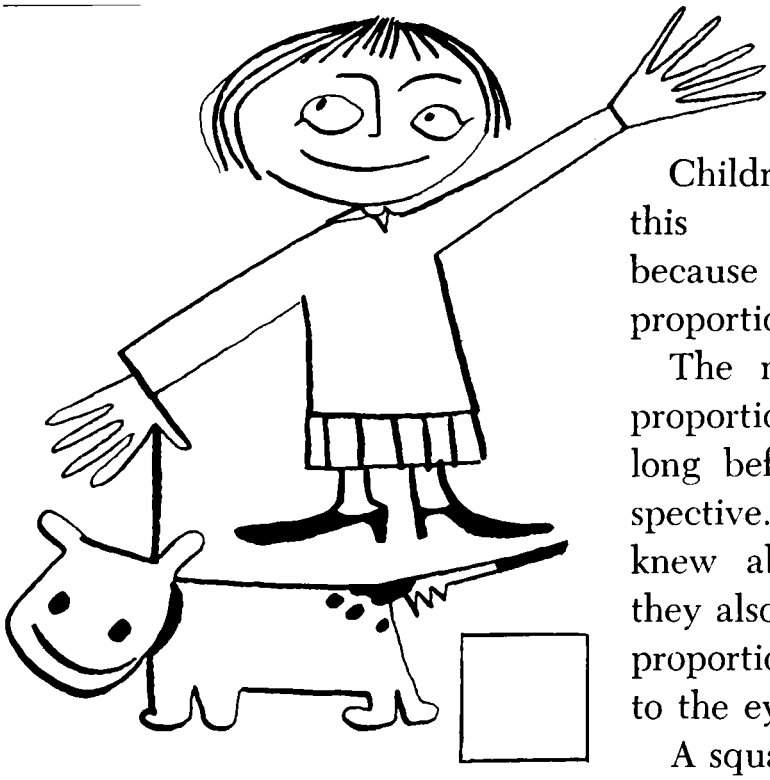


Later painters experimented with reducing shapes to their simplest forms: cubes or cones or triangles or circles. The mathematical shape of an object became more important than the real shape. A face became a series of angles, a bowl became a curve, a body became a square. This kind of painting is called *cubism.* The most important thing to the painter was the way the shapes were related.



Pablo Picasso was, for a time, fascinated with cubism

We said before that mathematics deals with relationships. In art, we call this relationship *proportion.* Proportion tells an artist how big to make a man standing next to a tree, how thick to make the legs on a chair, how long a man's arms should be when they are at his sides or stretched out.



Children make pictures like this
because they don't understand proportion.

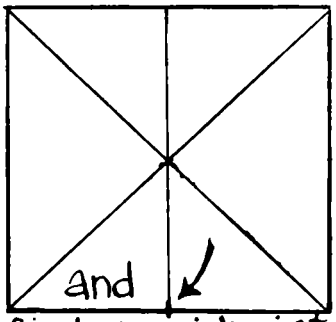
The mathematical laws of proportion were discovered long before the laws of perspective. The ancient Greeks knew about proportion, and they also discovered that some proportions are more pleasing to the eye than others.

A square looks squat.

But the Greeks discovered a rectangle so pleasing to the eye that they called its proportions the Golden Section.

How to construct a Golden Section rectangle

1. start with a square

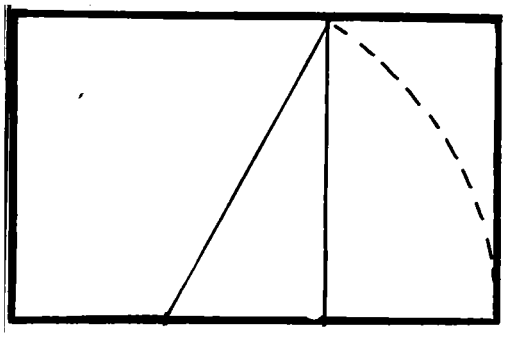


2. connect midpoint with corner

3. using that line as your radius, draw th arc with your compa

compass point here

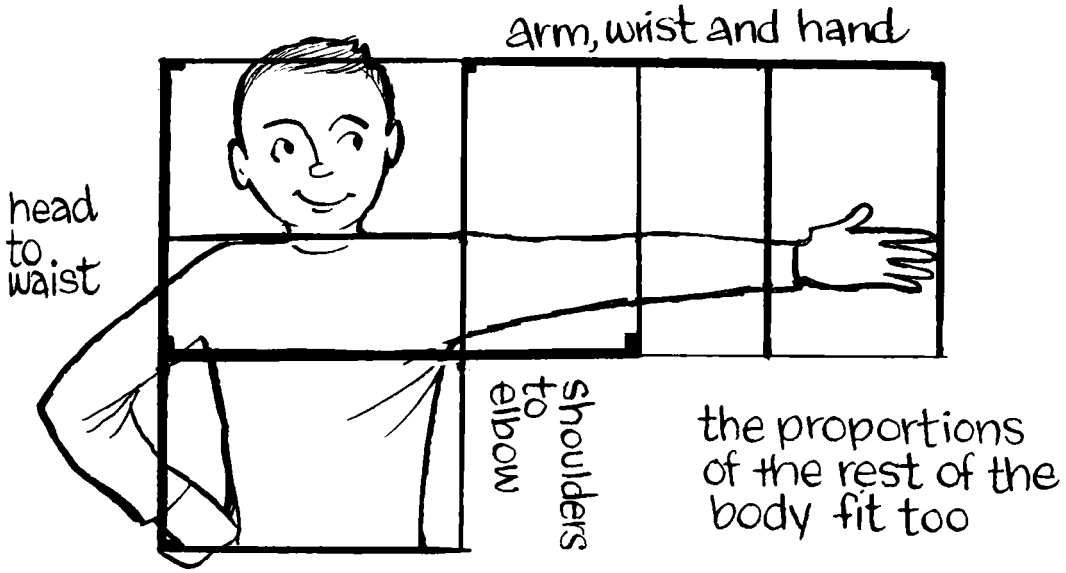
When the Greeks figured out the mathematical rule for the Golden Section, this is what they found:
The width and the length added together
are related to
the length alone
in the same
way that the length alone
is related to the width.



4. now finish the rectangle like this

Some modern artists feel that the proportions of the Golden Section are so beautiful that they can make exciting paintings using nothing else.

The proportions of the Golden Section appear in many places, even in the human body.



Breaking up the area of a canvas into pleasing divisions is one of the main aims of modern art. Often an artist will use purely mathematical shapes as a basis for his designs. But though covering a surface with interesting shapes is a very modern form of art, it is also one of the oldest. Indian pottery, Egyptian jewelry, Babylonian textiles and Greek vases were all covered with shapes to form designs.

Art—but mathematics too!

MATHEMATICS IN MUSIC

Have you ever stretched a rubber band out tight, and then plinked it? It almost sounds like music. When you stretch the rubber band tighter, or loosen it a little, you can make the tone higher or lower.

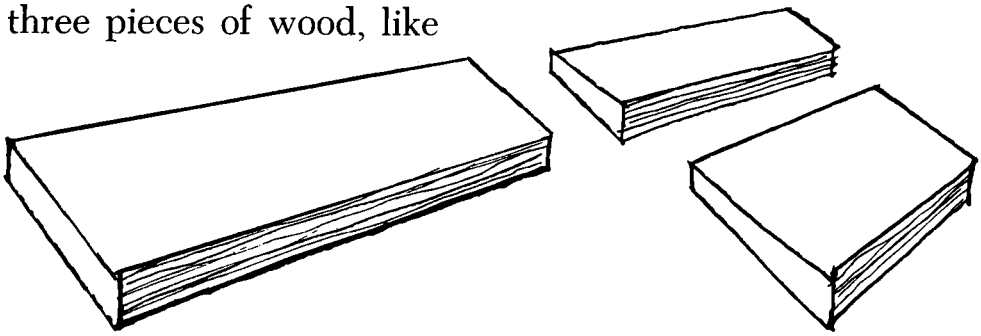
Are you making music on your rubber band? Well, not exactly. Music is made up not only of separate tones, but of certain special relationships between one tone and another. These relationships are called *intervals* and without intervals you can't have music.

Even from the very earliest times, when music was made just by voices or the simplest instruments, people found that certain intervals made the sounds more pleasant. But no one knew why.

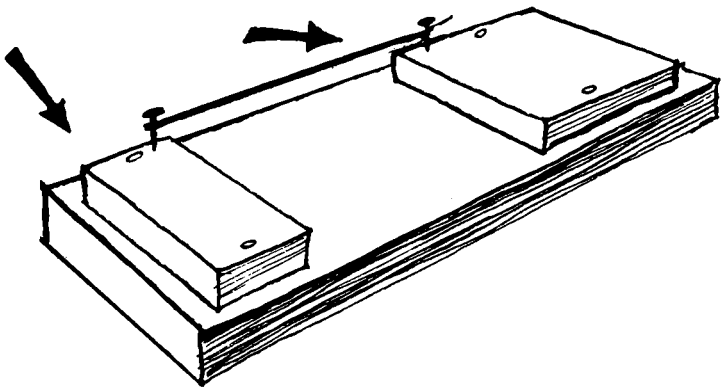
Then the ancient Greeks, who loved music *and* mathematics, began to think about the problem. First they learned what these "right" intervals were. We call them the octave, the fifth, the fourth, the major third and the minor third. These are all based on the "home tone," which is what the first string is called. When we sing, we often call the home tone "do."

Then the Greeks experimented until they discovered how these intervals were related. And how it delighted their mathematical minds!

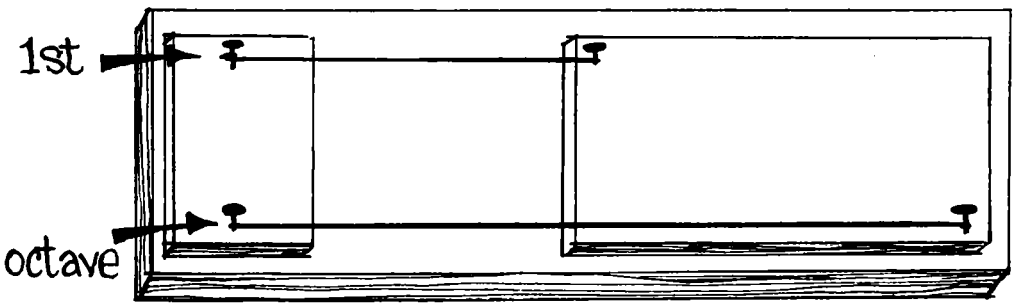
You can see these relationships yourself by making a kind of a musical instrument with six pieces of thin string, some nails, and three pieces of wood, like this.



First nail the two smaller pieces on the large board, like this



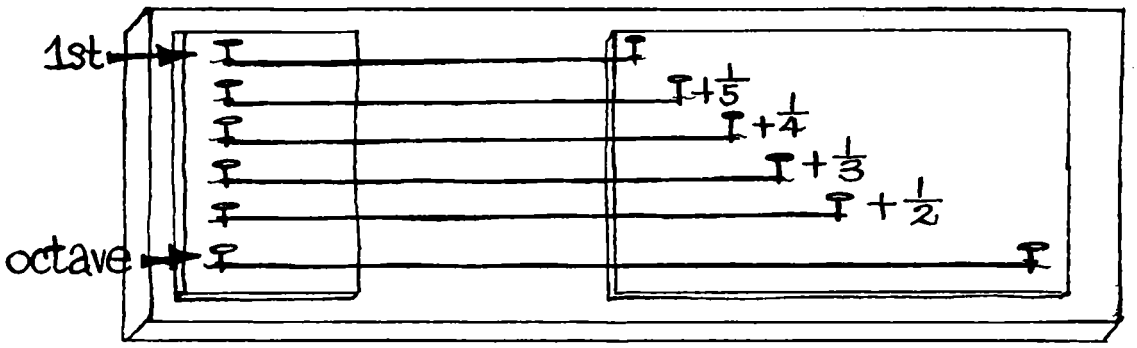
Make the first string any convenient size for your first note. This is your "home tone," or "do." Now put two more nails in the board and fasten the home tone string to them. (The picture shows you how it should look.) Make the second string exactly twice as long as the first one. This is your octave. It is the same note as the first string, but much lower. Now your board looks like this



The third string should be the length of the first string plus one half again. It goes just above the octave, and makes the interval called the fifth.

The fourth string should be the length of the first string plus one third. It makes the interval called the fourth. The next string up should be the length of the first string plus one quarter. It makes the interval called the major third. The next string is right below the first. It is the length of the first string plus one fifth. This is the minor third.

Now your sounding board looks like this:



these would be handy lengths for your strings

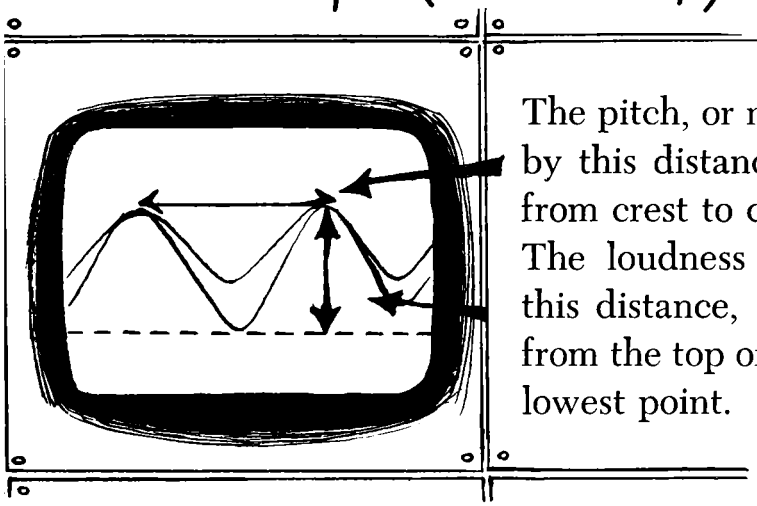
1st	5 inches
minor 3rd	6 inches
major 3rd	6 1/4 inches
4th	6 2/3 inches
5th	7 1/2 inches
octave	10 inches

the tighter the strings are, the better your music will sound

How beautifully mathematical these intervals are! Why are these sounds the ones that make our music? Certainly no one commanded that everyone sing or play this way. Therefore, the Greeks reasoned, here was one more proof that there is a definite mathematical order to the universe.

Music can be measured on a device called an oscilloscope. When an oscilloscope "hears" a tone, it translates it into a picture on its screen. The picture looks like a wave, or line of waves.

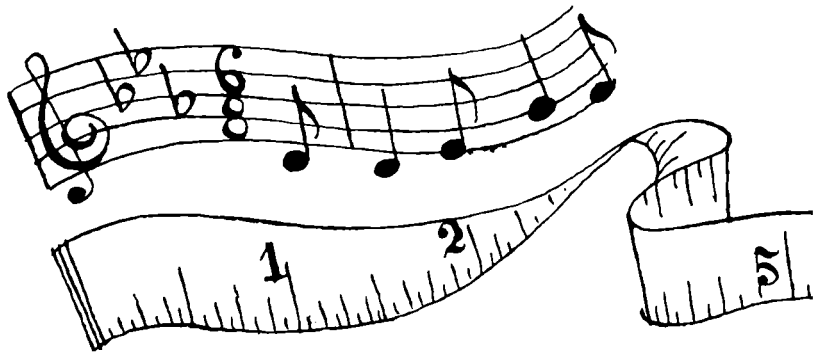
Oscilloscope (o sil o skop)



The pitch, or note, is measured by this distance, from crest to crest.
The loudness is measured by this distance, from the top of the wave to the lowest point.

the screen is only part of an oscilloscope. It contains many electronic tubes which change sound (or light) into electricity

So music can not only be heard, but we can see it and measure it mathematically.



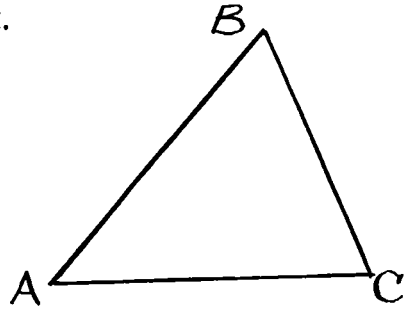
MATHEMATICS WITH LETTERS

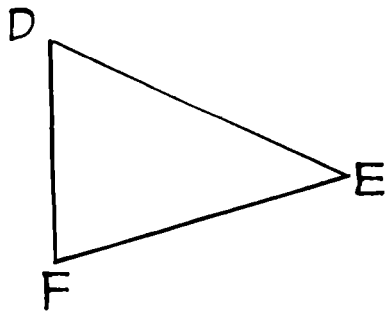
Using Letters for Labels

We labeled points on a line

to help us tell which point we were talking about.
We can name the whole line AE
or show that we are talking about a special part of the line by calling that segment AB, or DE, or whichever it is.

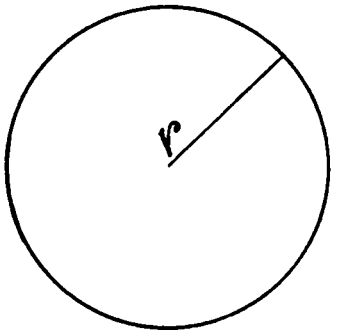
If we have two triangles, we label the angles, which mathematicians call *vertices* (ver' ti sees), and call them △ ABC (or △ CAB) and △ DEF. It doesn't matter where you start.

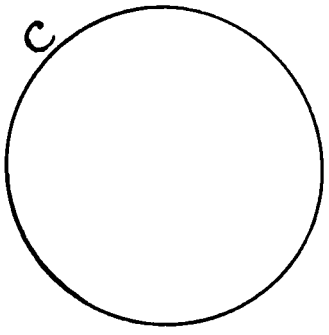




Sometimes we use letters to stand for numbers, but this is only done when everybody knows what possible numbers each letter stands for.

"r," in mathematics, may stand for the length of the radius of a circle. If we say "r = 12 inches" it means that the radius is 12 inches long.

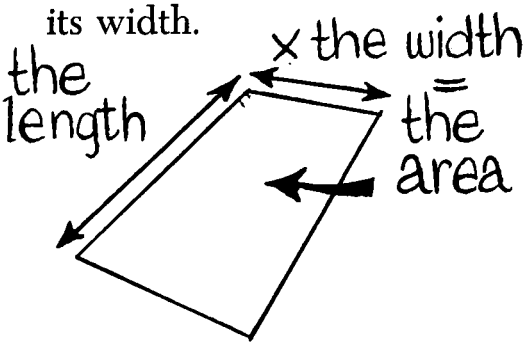




"C" in mathematics usually stands for the size of the circumference of a circle. We can say "C = 37 inches," which means that the circumference is 37 inches long.

Sometimes, in mathematics, we make whole sentences with letters.

You probably know that you can find the size (or area) of a floor or a field or any other rectangular surface by multiplying the measurement of its length by the measurement of its width.



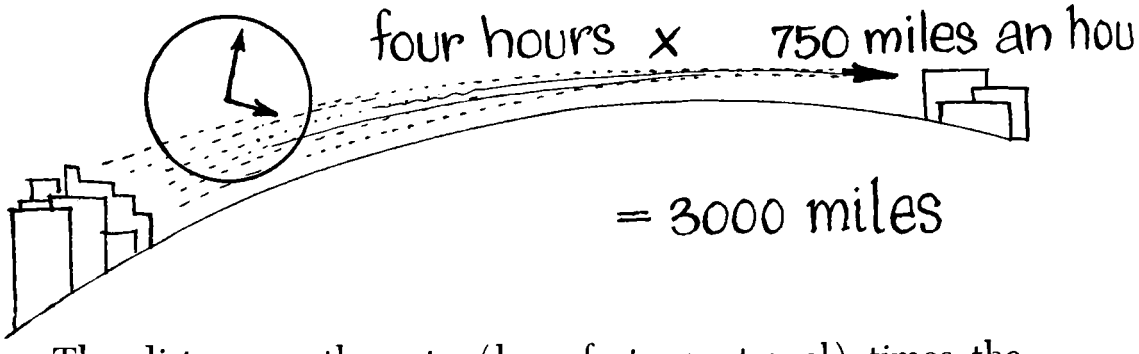
You could write it

$A = l \times w$ or $A = lw$

area = length x width

That's a whole number sentence without using any numbers!

If you know how fast a car or a train or a plane is going, and how long it takes to get from one place to another, you can find the distance between the two places.



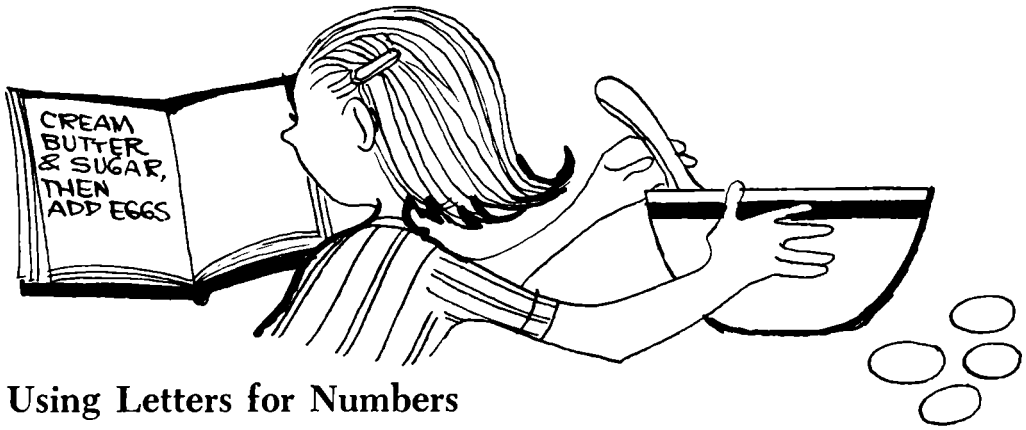
The distance = the rate (how fast you travel) times the time (how long you travel).

You can write it like this: $D = R \times T$ or $D = RT$

distance = rate x time

A sentence like this is called a *formula.*

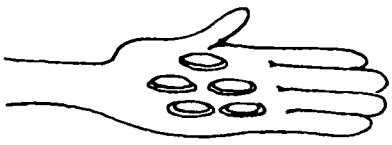
A formula in mathematics is like a recipe in cooking. They both tell you what ingredients you need to work with, and how to put the things together.



Using Letters for Numbers

In arithmetic we need numbers in order to do problems.

If you want to find out how many more pennies David has than Lisa, you have to know how many pennies David has, and how many Lisa has. Then you can say



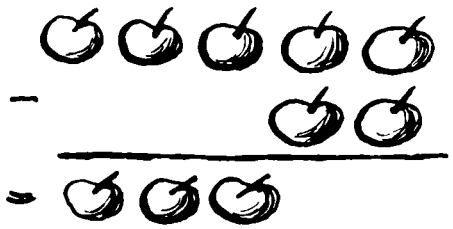
David has 5¢
Lisa has 2¢

$$\begin{array}{r} 5 \\ -\,2 \\ \hline 3 \end{array}$$

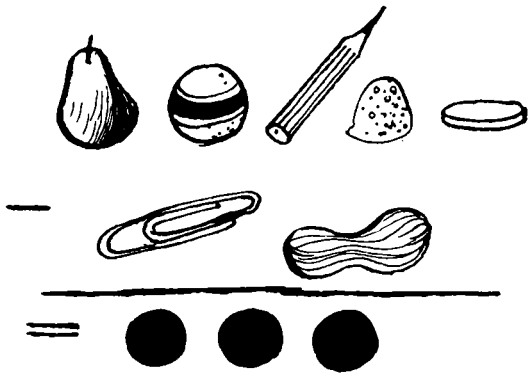
So David has three more pennies than Lisa.

If David had 5 apples and Lisa had 2 apples, you would do the same example

$$5-2=3$$



If you wanted to find the difference between 5 miles and 2 miles, you would do the same example, $5 - 2 = 3$



5 always stands for a certain number of things, no matter what the things are,

and 2 always stands for a certain number of things,

and the difference between those two numbers is aways 3.

But suppose we say that David has X pennies

and Lisa has Y pennies and the difference between them is Z pennies.

Now we are talking about *unknown* quantities. We don't know what numbers x or y or z stand for.

But we do know that $x - y = z$, because that is always the way we find the difference, no matter what the actual numbers are. How many sets of numbers can you find which will make $x - y = z$ true?

We could try

$$x - y = z \quad \text{so} \quad x = 5,\ y = 2,\ z = 3$$
$$5 - 2 = 3$$

or we could try

$$10 - 6 = 4 \quad \text{so} \quad x = 10,\ y = 6,\ z = 4$$

or $100 - 80 = 20$ so $x = 100,\ y = 80,\ z = 20$

And they would all show that $x - y = z$ is true for these sets of numbers.

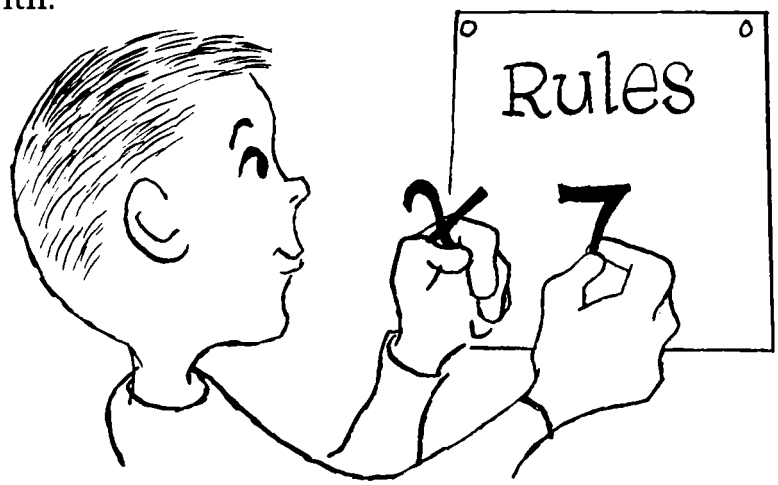
But suppose we said that $x = 10,\ y = 20,\ z = 30$

Then is $x - y = z$ true? Of course not.

$$z - y = x$$

We usually use the letters x, y and z to stand for numbers we do not know. There's no real reason for this; it's just something that most mathematicians do.

All mathematicians—and this includes you—use rules to tell them what they can do with the numbers or symbols they are working with.



Suppose we wanted to make a rule to show that if you add two numbers together, it doesn't matter if you add the first number to the second, or the second number to the first. That's something we all know, but hardly ever think about.

$3+2=2+3$

or

$10+20=20+10$

or

$1492+1776=1776+1492$

Right? Of course. We know that whether we put the 2 first or the 3 first, the answer is always 5.

The answer to both is 30.

It doesn't matter which is first, does it?

We can make a mathematical rule about this fact using letters:

This really says that the order in which you add two numbers doesn't matter; the answer is the same. So $a + b = b + a$ is an important mathematical fact that uses no numbers.

How about $a \times b = b \times a$

Is that true? Let's try it by substituting the numbers for the letters.

$6 \times 4 = 4 \times 6$ The answer on both sides is 24.

$27 \times 56 = 56 \times 27$ Is that correct? Multiply it and see.

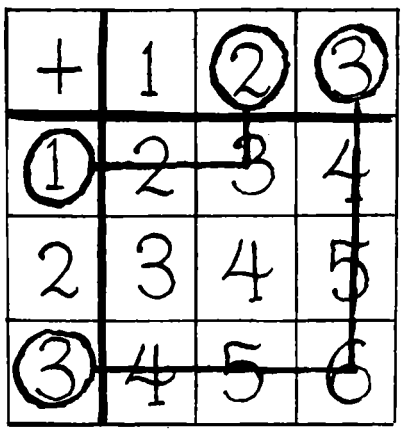
If it is, we can say that for any numbers a and b, $a \times b = b \times a$.

New Number Systems Using Letters

Do you know what this chart says?

+	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

It is an addition chart, and it tells us the answers to all the addition facts up to $3 + 3 = 6$. Wherever the lines from any two numbers intersect, you find the answer.



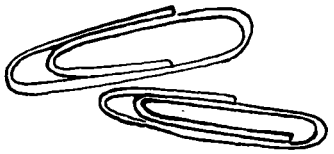
We could make the chart much larger; up to $10 + 10$ or $100 + 100$, or as far as we wanted to go, and it would always give us the answer.

Now, can you tell what this chart says?

This is an addition chart too. This letter chart goes up to c + c = f. Use your imagination and try to go up to d + d, or e + e. Copy the chart below, and see if you can fill in the blanks.

+	a	b	c
a	b	c	d
b	c	d	e
c	d	e	f

+	a	b	c	d	e
a	b	c	d		
b	c	d	e		
c	d	e	f		
	e		g		
e					



You could even make up a new number system, like this

+	□	○	△			
□	○	△	∩			
○	△	∩	Z			
△	∩	Z	●			

Can you make this chart bigger too?

Use any symbols you want to.

The important thing to remember is that when any two numbers, or the letters, or symbols that stand for them, are added together, there can only be *one* right answer.

But it doesn't matter if you say

$1+2=3$ or $2+1=3$ (Check it on the number chart)

You can say

$c+a=d$ or $a+c=d$ (Check it on the letter chart)

You can say

$\square+\bigcirc=\triangle$ or $\bigcirc+\square=\triangle$ (Check it on the symbol chart)

So

$$1+2=2+1$$
$$c+a=a+c$$
$$\square+\bigcirc=\bigcirc+\square$$

The fact that it doesn't matter in which order you add two things is one of the basic rules of mathematics. It is called the *commutative property of addition.* (To *commutate* is to change the order of things.)

Is there a *commutative property of multiplication?* Certainly. We showed that $a \times b = b \times a$, and that's what it means.

Here is a multiplication chart showing the commutative property.

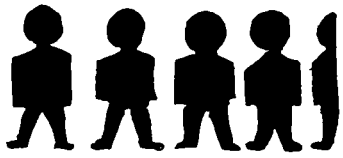
×	1	2	3
1	1	2	3
2	2	4	6
3	3	6	9

×	a	b	c
a	a	b	c
b	b	c	
c			

can you finish this chart?

And if you're very ambitious, you can make up a multiplication chart using symbols too.

MATHEMATICS WITH CHARTS



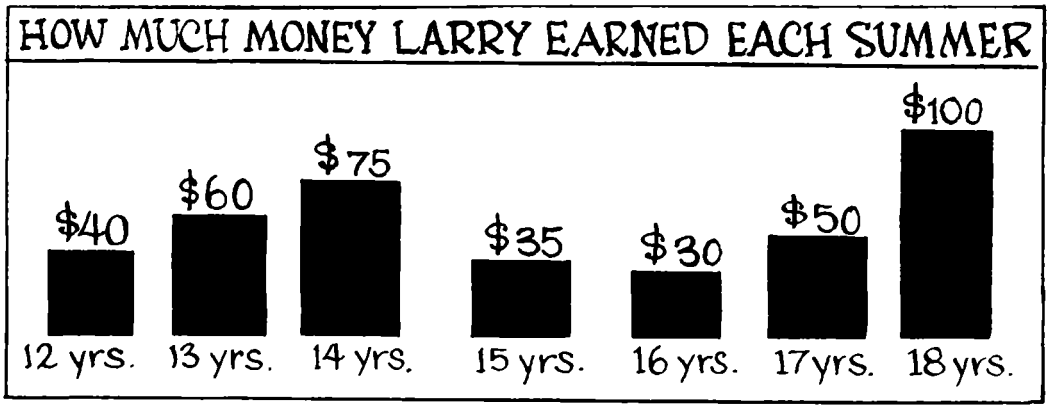
Graphs

A graph is a way of telling a story with pictures. Remember how we wrote number sentences with letters? Well, we can write a whole number story with a graph.

The word *graph* comes from the Greek *graphein,* which means to draw or write. Autograph, telegraph, photograph—all have something to do with pictures or writing.

But you can't read the number story written in the graph unless you know the title. The title tells what the story is about.

Probably the easiest kind of graph to read is a bar graph. What story can you tell from this one?



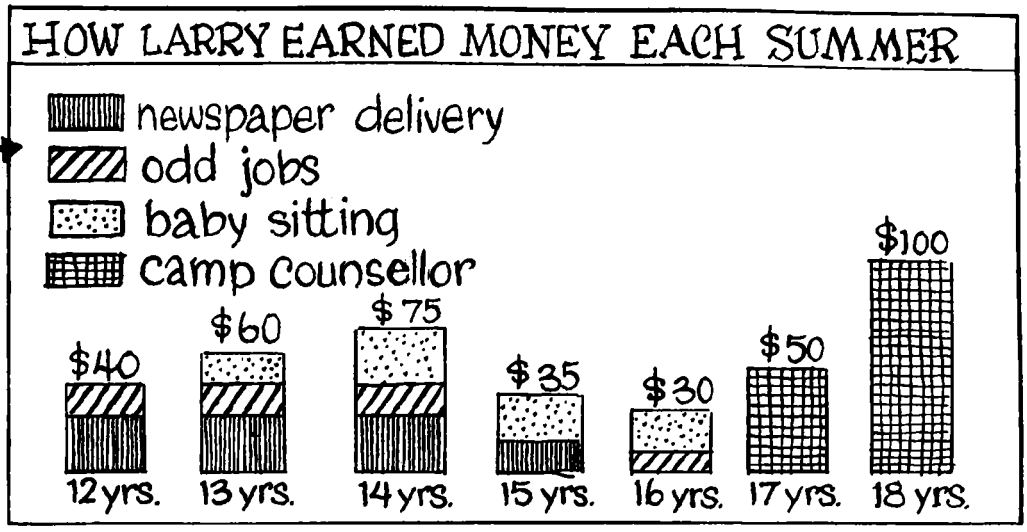
How old was Larry when he earned the largest amount of money?

How old was he when he earned the least?

When did his earnings fall off sharply?

Maybe this bar graph tells you all you want to know about how much Larry earned each summer. But suppose you're more curious. *Why* are the amounts so different?

To find out, we can use a more complicated kind of bar graph. It will be almost like solving a mystery, but like any good mystery, there must be a key. (You will usually find the key in the corner of the graph.)



Now what have you learned about Larry? Isn't the story clearer when you can see how he earned his money?

When did he stop delivering papers?

What happened when he stopped?

What happened to his earnings when he was seventeen and eighteen years old? And what happened to his summers, besides? Do you think the story has a happy ending?

Line Graphs

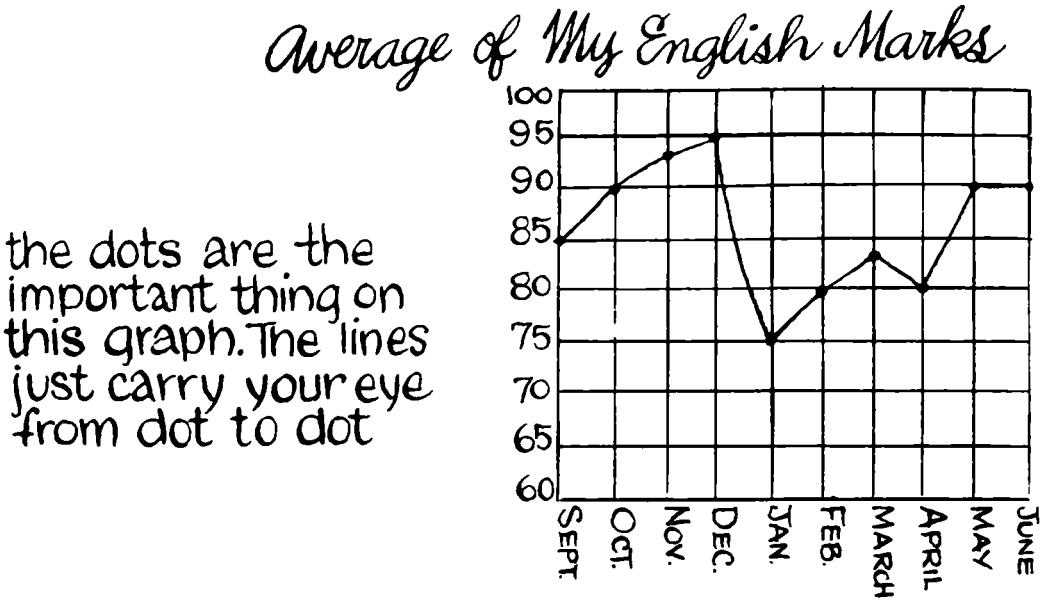
Have you ever seen a piece of graph paper, covered with lines that form little squares? This paper is used for making line graphs. To read them, you need three pieces of information.

1. What is the title?

2. What is written along the bottom line of the graph? (The bottom line is called the *horizontal axis.* It usually tells the time when the story happens.)

3. What is written up the side of the graph? (This is called the *vertical axis,* and it usually tells the characters of the story —number of people, amount of money, kinds of candy.)

To read a line graph, start at the first place where there is a mark on the left side of the graph. This tells you *when* something happened, and *what* happened. Then follow the line to read the story.



What mark did this girl get in English in September?

Did she get better or worse as the term went on?

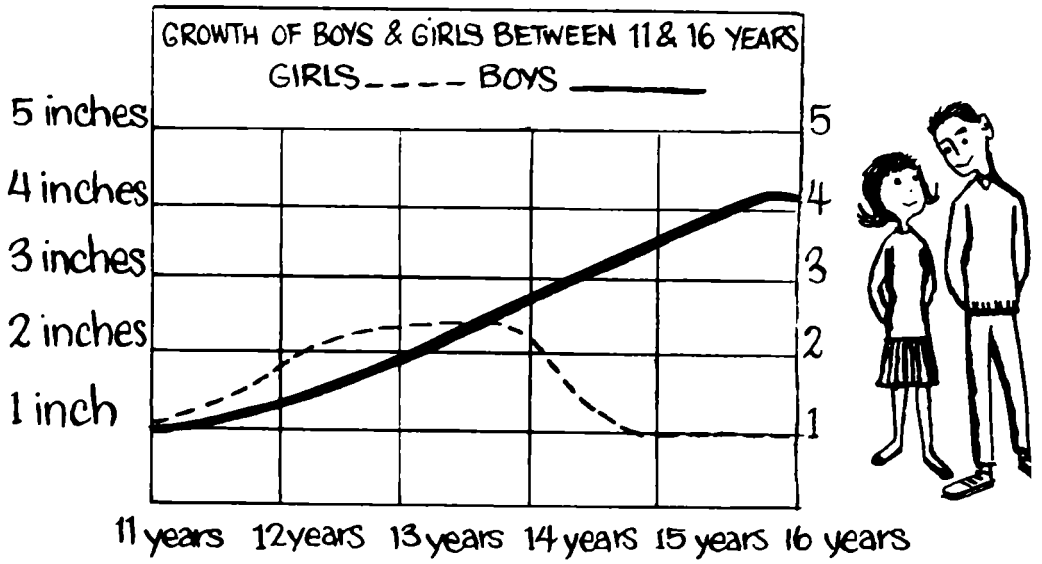
What happened when she came back after the Christmas holiday?

You can tell the whole story just by looking at this graph!

What kind of story would your English marks make if you graphed them?

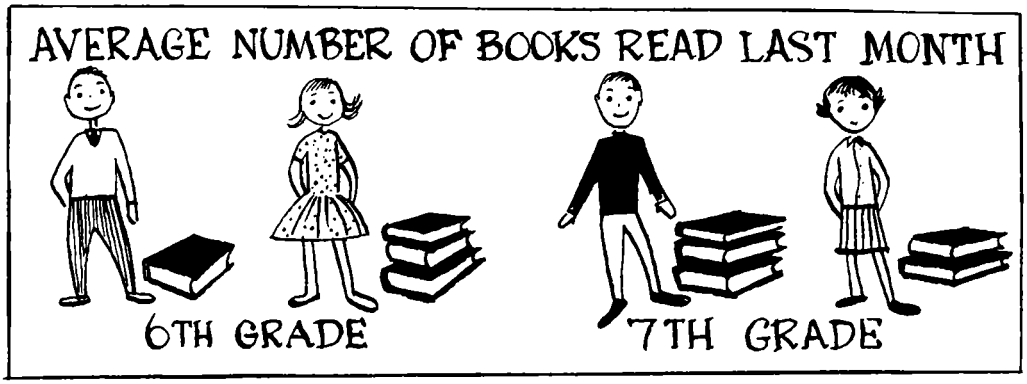
You can make a record of the temperature with a line graph, or show how much taller you get each year, or how your town or city grows.

You can also chart two or more related things on one graph, so you can compare them. The key will tell you what each line stands for.

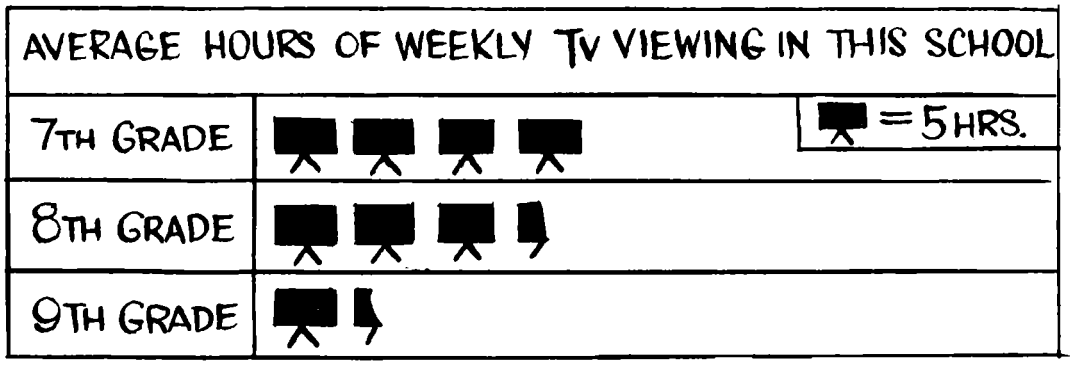


Pictographs

Sometimes charts are made almost entirely of pictures. These pictographs need very little explanation.

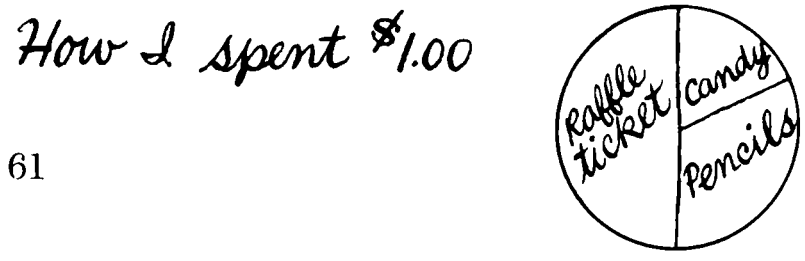


Another way of making a pictograph is to combine it with a bar graph. Pictures are used instead of plain bars, and there is usually a key.



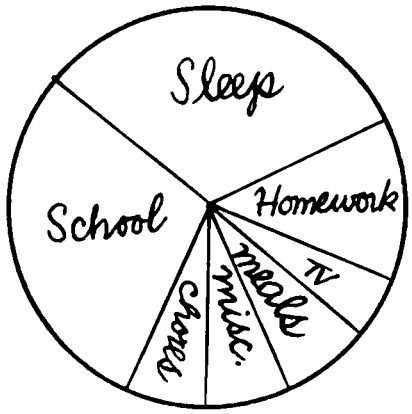
Circle Graphs

A circle graph is often called a "pie chart." It shows the relationships of the sizes or amounts of things by showing them as "slices of pie." The important thing to remember about a circle graph is that the different parts must make up the *whole* of something—all of your money, or all of the products of a country, or all of the time spent.



When you add everything together, remember, it must add up to the whole dollar.

How do you spend an average day? If you made a circle graph of it, maybe it would look like this

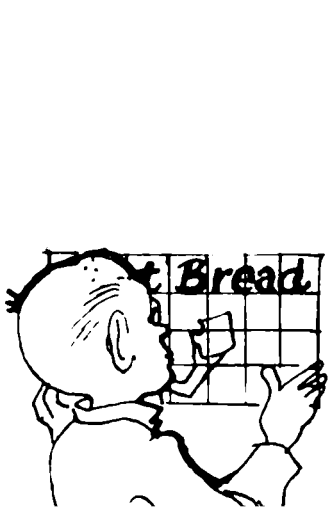


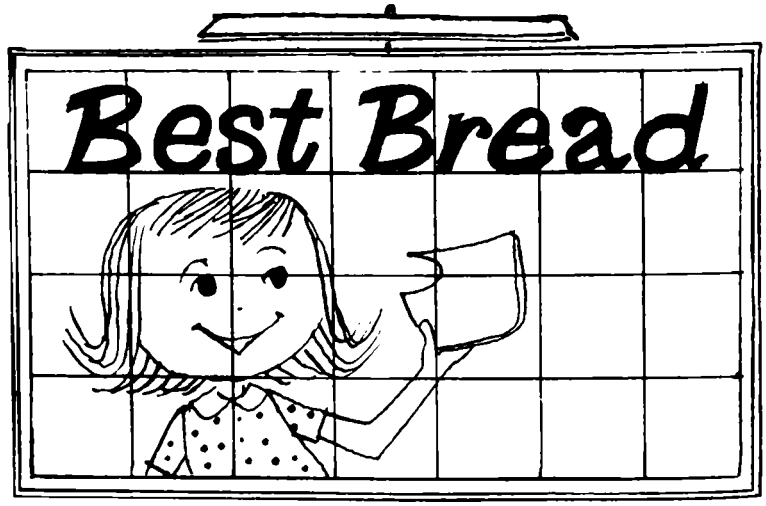
You have to put the "miscellaneous" in, even though you're not sure exactly how you spent that time, because that's what is left of the twenty-four hours after eating, sleeping, studying, chores and television. The whole circle must show the whole day. Maybe you can put more "slices" in your chart.

Scale Drawings

Graph paper is not only used for number graphs. Sometimes it is used to draw something larger or smaller than the original. It can also be used to make an exact copy.

Have you ever seen men painting a billboard? This is how they do it.

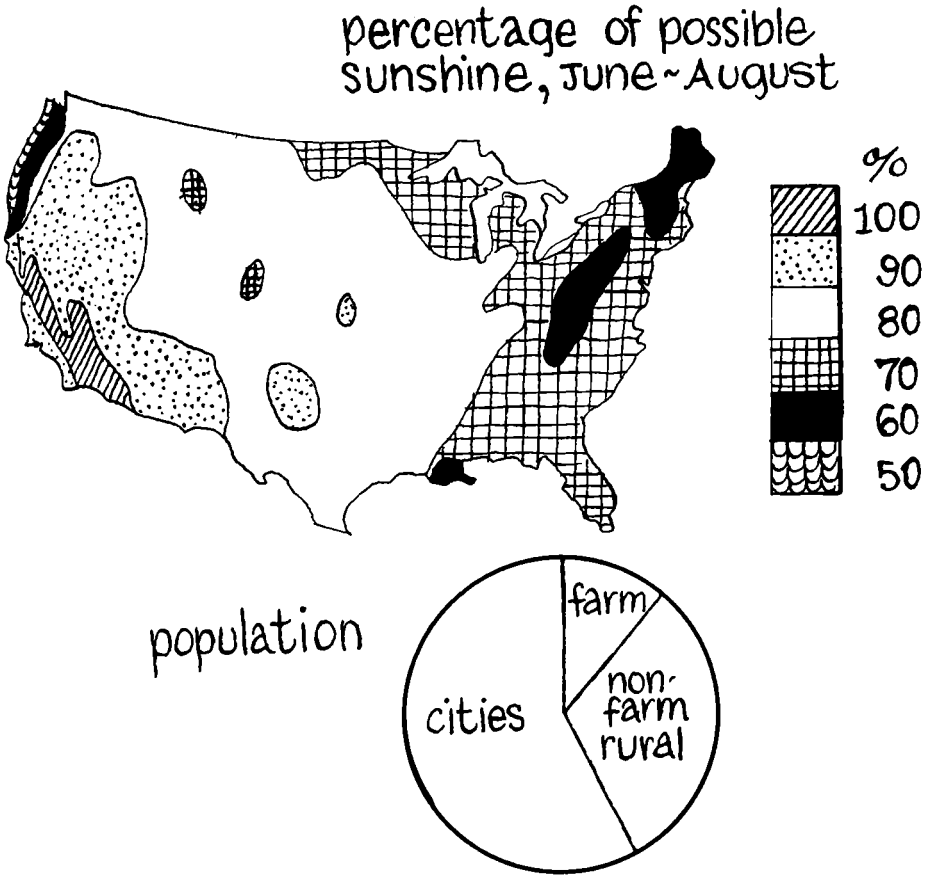




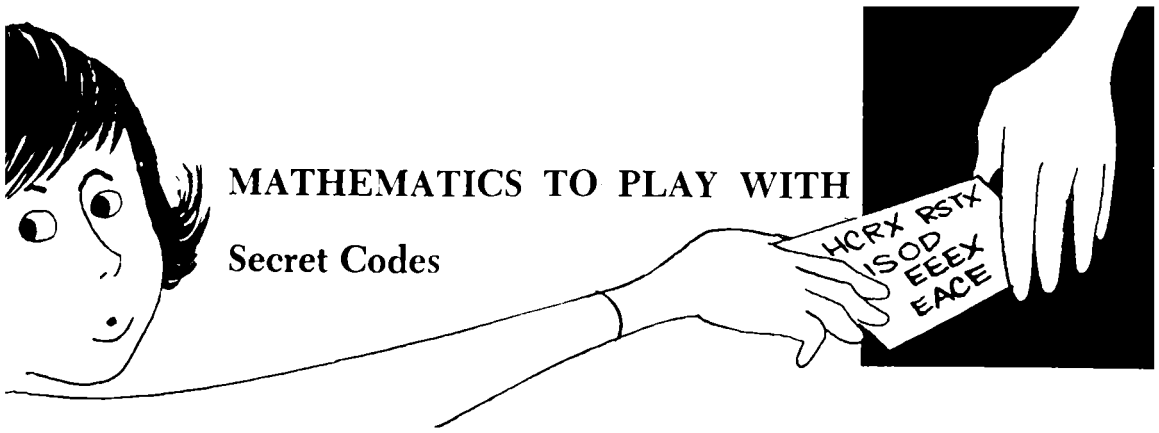
Carpenters and knitters use scale drawings too. A room plan is a scale drawing, and so is a blueprint.

Probably the scale drawing most often used is a map. Mapmakers reduce things of immense proportions, like a city or a country or a continent, to a size that will fit on a sheet of paper or a page in a book. And still, maps are so accurate that we know we can depend on them whether we are studying or traveling.

You can see many other things besides distances and land shapes on maps.
Things like these



And you don't need numbers—
just the key in the corner.



MATHEMATICS TO PLAY WITH

Secret Codes

We use keys on graphs so that everyone will know what the graph means. But the key to a code has to be a secret. Unless you have the key, you won't know what the message is.

There are two main ways for making up secret codes.

The first is *substitution*.

We simply substitute something for each letter.

We might substitute another letter, like this

A	B	C	D	E	F	G
Z	Y	X	W	V	U	T

(using the alphabet backward as a code.)

Or we can substitute numbers, like this

A	B	C	D	E	F	G
1	2	3	4	5	6	7

or even make up our own signs, like this

A	B	C	D	E	F	G
□	✳	⬮	⧖	△	▬	◪

One of the most famous substitution codes is used in a story about Sherlock Holmes. Here is part of the first line.

(If you want the answer you'll find it in "The Adventure of the Dancing Men.")

Another way of substituting is by using a "square code." If you leave X out of the alphabet, there are 25 letters, which can be written in a square, like this

	1	2	3	4	5
1	a	b	c	d	e
2	f	g	h	i	j
3	k	l	m	n	o
4	p	q	r	s	t
5	u	v	w	y	z

Do you remember the way we did addition and multiplication squares? The place where the lines met was the answer. The square code works almost the same way. Suppose we want to write C, which is the first row, third column. We write 13. Every letter has two numbers that substitute for it.

Can you figure out this word? **13351415***

What other kinds of substitution codes can you think of?

Transposition is usually an easier way to make a code than substitution is, and it is even harder to solve. Transposition means "changing places." The important thing is to have an orderly way to change the places of your letters.

Watch how it is done here.

My name is John

My name
is John

M	y	n	a	m	e
i	s	J	o	h	n

Miysnjaomhen

But suppose you don't have an equal number of letters in each line? You can always add X's! And you can divide the message into as many lines as you want to. Try to make the shape of the message as close to a square as possible.

HERE IS A SECRET CODE

H	E	R	E	I
S	A	S	E	C
R	E	T	C	O
D	E	X	X	X

7 letters, so let's add 3 more to make 20.
0 can be divided into 4 lines of 5 letters each.

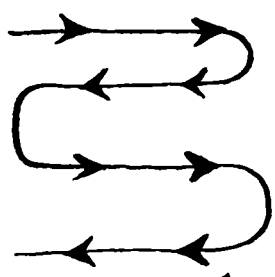
*13351415
C O D E

How would you write the message?

HSRD EAEE RSTx EECX ICOX

You can do all sorts of things to make the code harder. You can write the lines like this

H	E	R	E	I
C	E	S	A	S
R	E	T	C	O
X	X	X	E	D



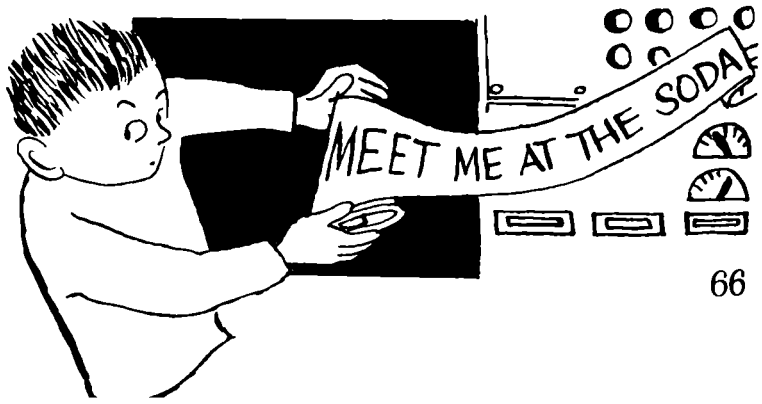
1	2	3	4	5
H	E	R	E	I
C	E	S	A	S
R	E	T	C	O
X	X	X	E	D

You can put the "scrambled" message in a different order. Instead of writing the columns 1,2,3,4,5, you can use 1,3,5,2,4

HCRX RSTx ISOD EEEX EACE

Can you make up a code combining substitution and transposition?

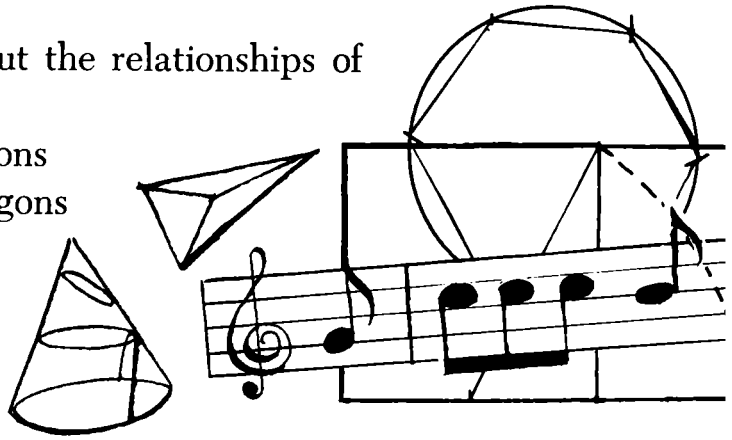
Once governments used to have expert "code breakers" to solve codes. Now computers do the job. The interesting thing is that codes are always "broken" mathematically. This is because a computer can work its way so swiftly through all the possibilities. But no matter how complicated a secret code is, it usually makes use of substitution and transposition.



SPATIAL RELATIONSHIPS

We said that mathematics is a special way of thinking. And we said that mathematics is the science of relationships.

We have talked about the relationships of
lines in angles
curves in conic sections
shapes in many polygons
planes and solids
sizes in art
intervals in music
bars in graphs
and more.

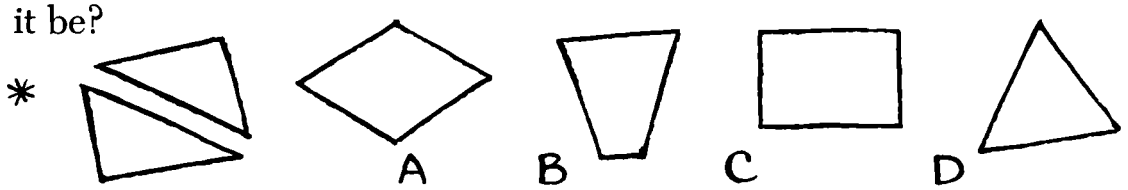


There is one kind of mathematical thinking that relates *parts* of shapes.

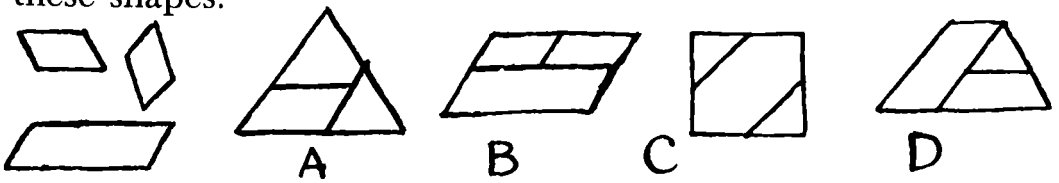
How well can you picture objects in space, and move them around in your mind?

In each of the following puzzles, the pieces must be fitted together without any gaps or overlapping. You can turn them around or turn them over to make them fit. But you must move or turn the pieces in your mind!

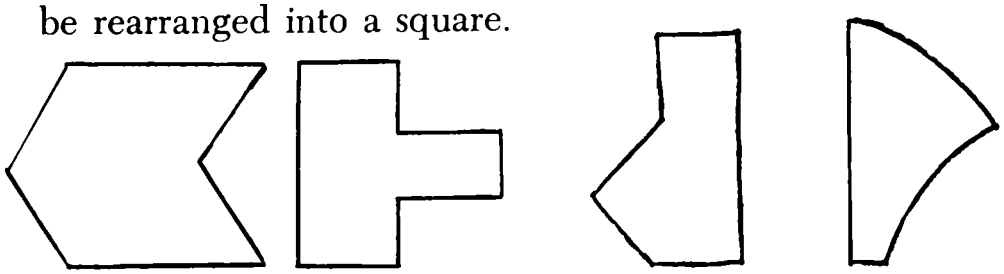
Look at the first two pieces. You can put them together to make only one of the following shapes. Which one would it be?



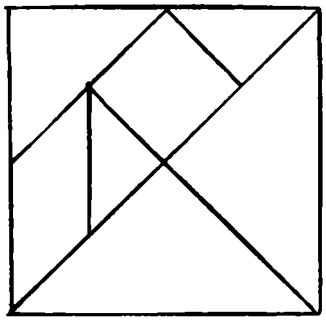
Here are three pieces. Fit them together to make one of these shapes.



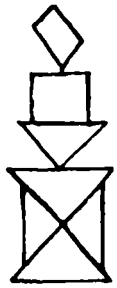
Each of the next four figures can be turned into a square by making one cut with a pair of scissors. (Cut them out of cardboard, or draw a dotted line to show where you would cut.) The cut divides each shape into two pieces which can be rearranged into a square.



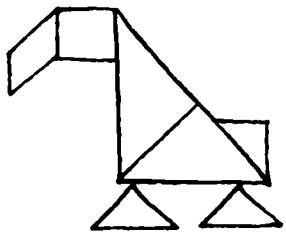
Almost four thousand years ago the Chinese made puzzles by dividing a square into seven pieces. Many books have been written about these puzzles, and people still enjoy them. The square is divided like this



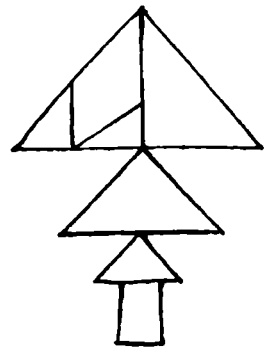
It is called a *tangram.* Hundreds of pictures can be made with these seven pieces. You can draw the pieces, but it's even more fun to cut your own tangram out of paper. You must use all seven pieces each time.
Here are a few ideas:



candlestick

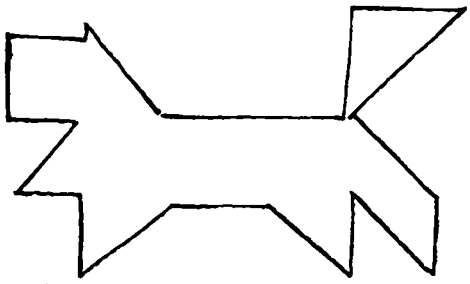


duck

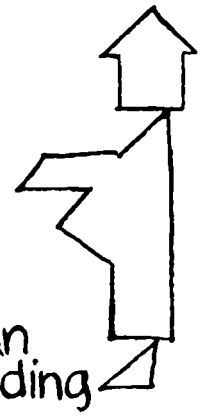


tree

There is another way to use a tangram. Make a picture and just show the outline. See if anyone can figure out how you used the pieces to make the picture.
Here are a couple for you to try:



dog running

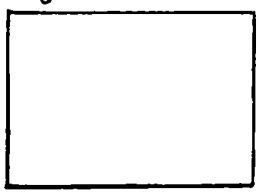


man standing

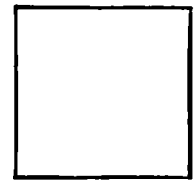
Here is a problem that seems simple, but you must have a very good sense of spatial relationships to solve it.

Can you cut this rectangle into only two pieces, and move the pieces around to form a square?

change this

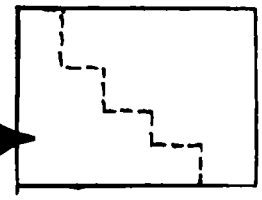


to this

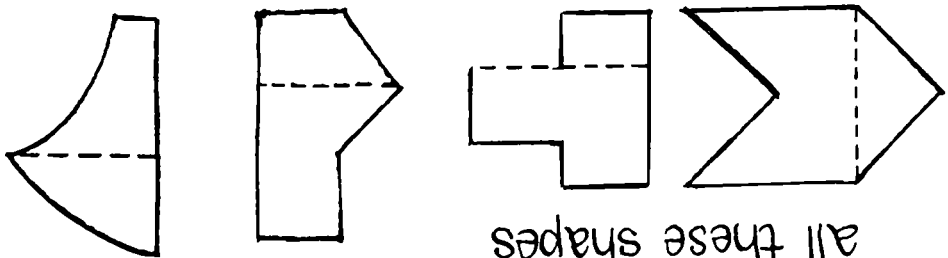


turn the book upside down to find the answers to the cutting puzzles

slide this piece under



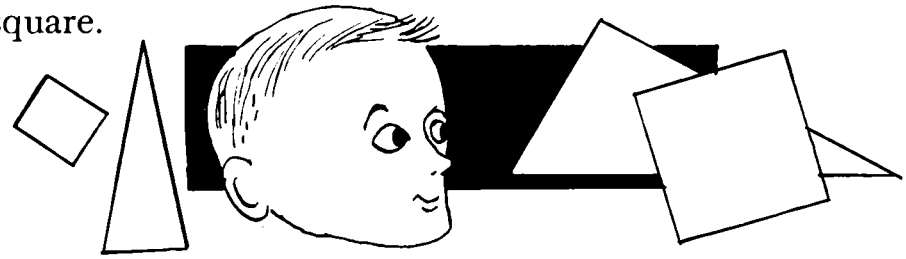
cut along the dotted lines to make squares of all these shapes



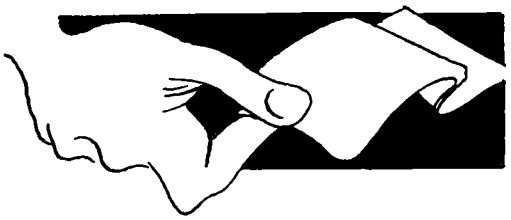
TOPOLOGY

We have just been moving figures in space to form different figures, or to show how the figures are related.

But did you notice something about all these figures? When you moved them, they always kept their shapes. No matter which way you turned a triangle, it was always a triangle. No matter how you turned a square, it was always a square.



But sometimes when you move a figure it doesn't remain the same.

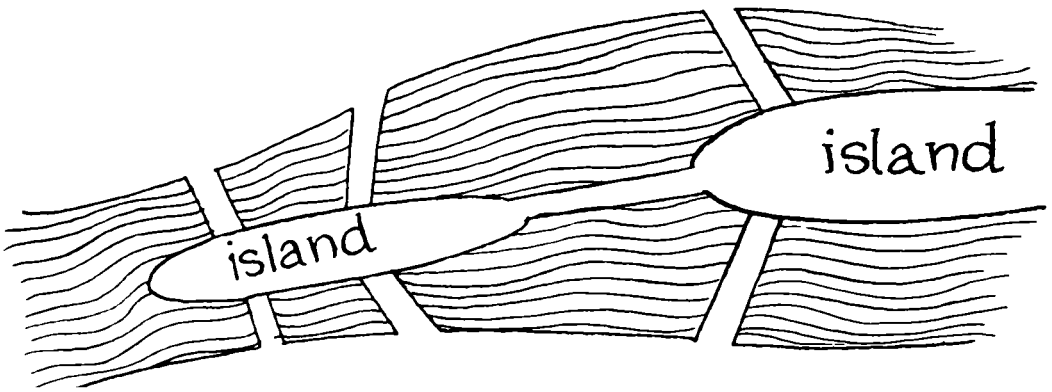


What happens when you hold up a rubber band?

What happens when you wave a piece of paper in the air or twist a paper clip?

Mathematicians hadn't really thought of shapes as changing until about two hundred years ago, when a mathematician named Leonard Euler became interested in solving a puzzle about bridges.

At first it seemed simple. There were seven bridges in the town. Was it possible, in the course of a walk, to cross all seven bridges without recrossing any of them?



Euler worked and worked at the problem. After a while he realized that the size of the bridges was not important,

measurement was not important,

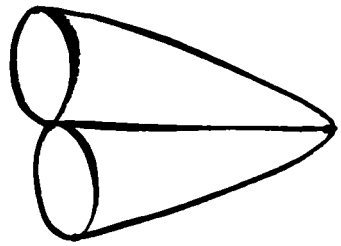
shape was not important,

and neither was distance.

Angles, curves and speed were not important.

The only thing that *was* important was *position.* Everything depended on the *positions* of the bridges, and on the *positions* of the lines he drew to represent them.

can you see that these lines represent the bridges? Their positions are the same



Could he draw these lines without retracing any of them?

Could he cross all the bridges without retracing his steps? The answer is no. It can't be done.

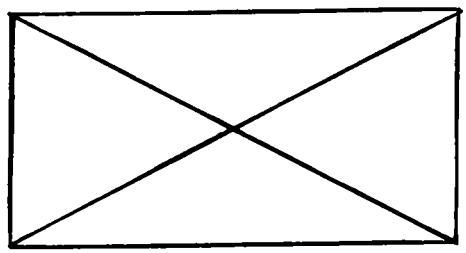
This was the beginning of a completely new kind of mathematics called *topology* (to pol' e jee).

Topology is sometimes called the science of direction, or the mathematics of position. The important thing in topology is not how far you have to go, but which way you have to go to get there.

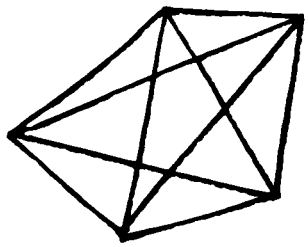
Topology is filled with puzzles that are fun to do, but they were all invented by very serious mathematicians.

For instance:

This figure can not be made with one pencil stroke. You must either lift your pencil from the paper or retrace a line.



But you won't have any trouble making this much more complicated figure with one pencil stroke.

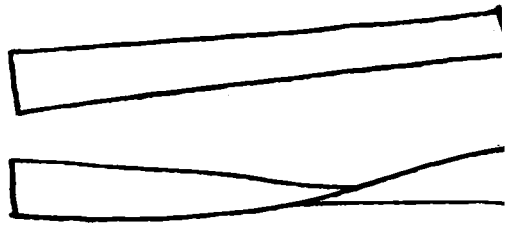


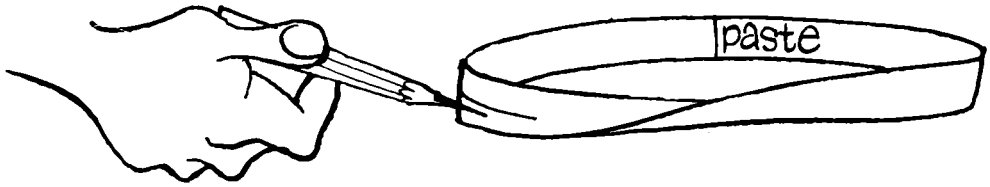
August Ferdinand Moebius was another mathematician who became interested in topology. He invented the Moebius strip, a piece of paper with only one side.

But everyone knows that a piece of paper has two sides! How could you possibly prove that a piece of paper has only one side?

Look at this.

Take a strip of paper, give it a half twist, and paste the ends together to form a ring.

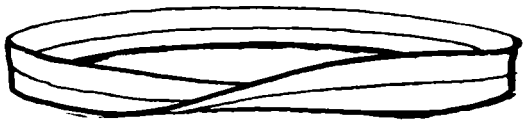


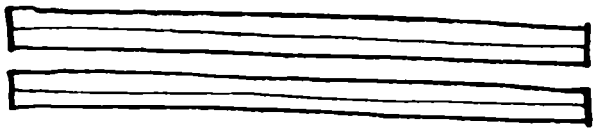


But certainly there is an outside and an inside to the ring!

Now start a pencil line at any point on the strip and keep drawing, without ever lifting your pencil, until you get back to your beginning point.

Open the pasted ends and lay the strip flat. Turn it over. Both sides have the pencil line, even though you only made one line. You have proved that the Moebius strip has only one side.





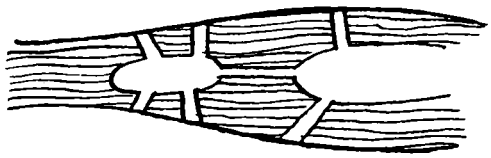
there will be a pencil line down each side of the strip

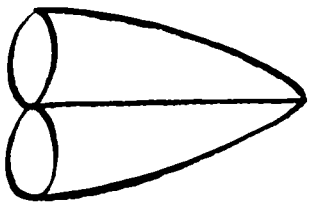
Here is another interesting puzzle in topology. How can a man take off his vest without removing his coat?

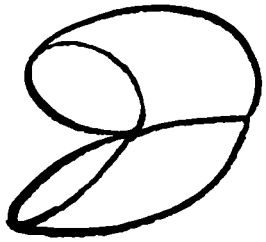
Does that sound like mathematics?

By now you know that shapes don't have to be rigid and unchanging.

When Euler drew these bridges like this







or even if he had drawn them like this

the problem would have still been the same.

The shapes of things *can* change, and that should give you your key.

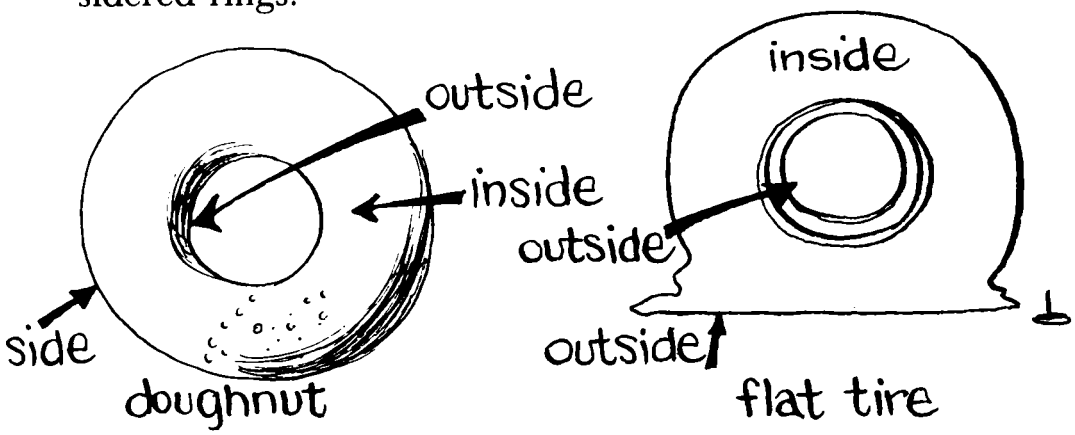
The man unbuttons his vest. (But it's still a vest, isn't it?) With his right hand he pushes the vest as far down on his left arm as he can. (Once he has his elbow through the armhole, the problem is just about solved.)

He pushes the vest all the way down, so his left arm is out of it. Then he pulls the vest through the right sleeve of his coat, and there it is!

Topology concerns position and direction.

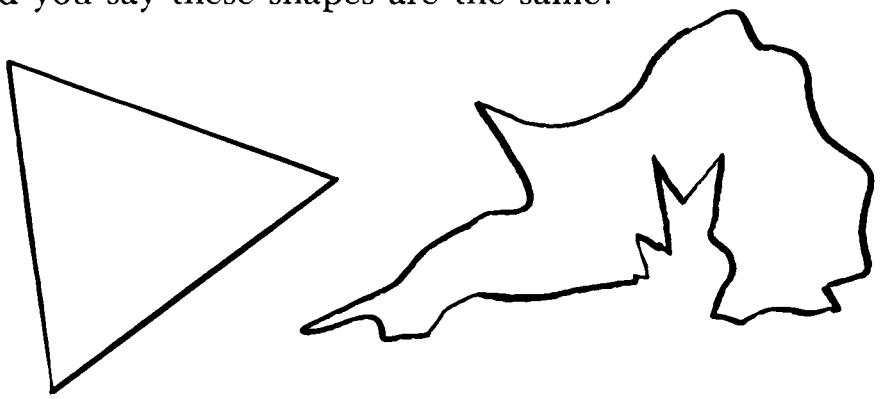
It is also about insides and outsides.

Sometimes people say that topologists can't tell a doughnut from a flat tire. That is because both things are considered rings.



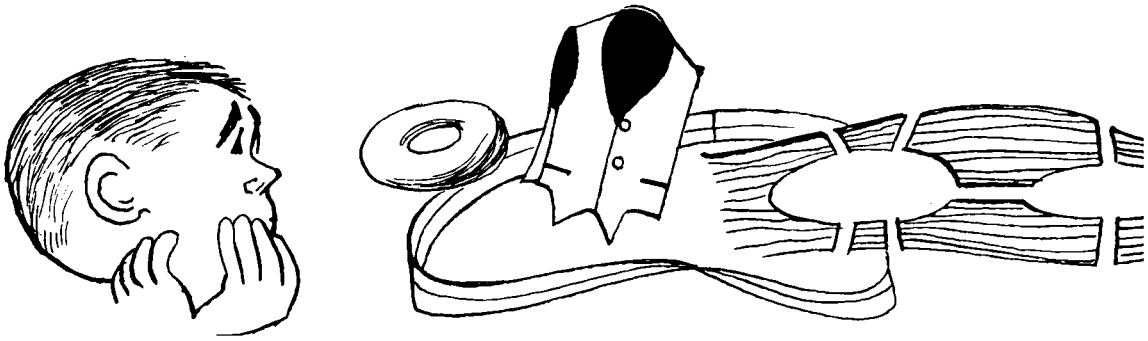
Both things have two outsides and one inside, so a topologist says they are the same.

Could you say these shapes are the same?



You could, if you were a topologist. They each have one inside and one outside, and that would be the important thing.

Why do mathematicians study and wonder about such strange things? What difference does it make whether a figure has one or two or ten insides or outsides? Why do mathematicians try to solve puzzles about bridges or vests?



Many of the answers they have found have already been put to work. Topology helps in the complicated programming of computers. Topology is at work in industry and atomic science and astronomy.

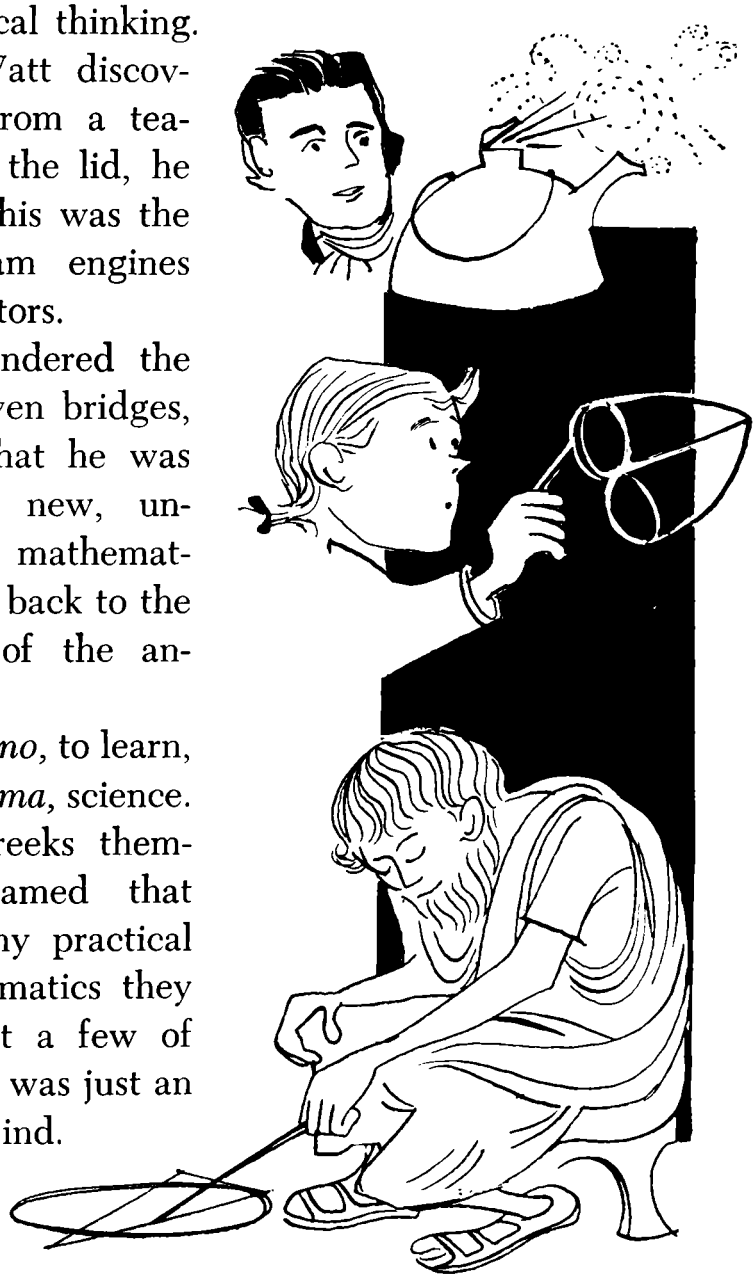
But even more important, topology has opened new horizons in mathematical thinking.

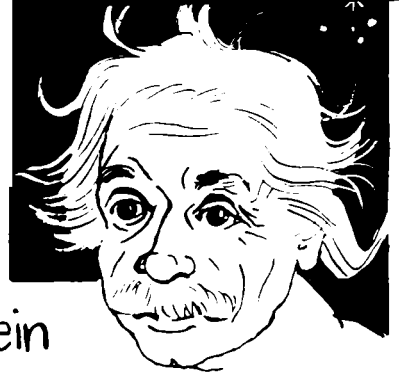
When James Watt discovered that steam from a teakettle could move the lid, he had no idea that this was the beginning of steam engines and all sorts of motors.

When Euler pondered the problem of the seven bridges, he had no idea that he was starting a whole new, unthought of field in mathematics. He was going back to the original meaning of the ancient Greek words

manthano, to learn,
and *mathema,* science.

Most of the Greeks themselves never dreamed that there would be any practical use for the mathematics they loved. To all but a few of them, mathematics was just an adventure of the mind.

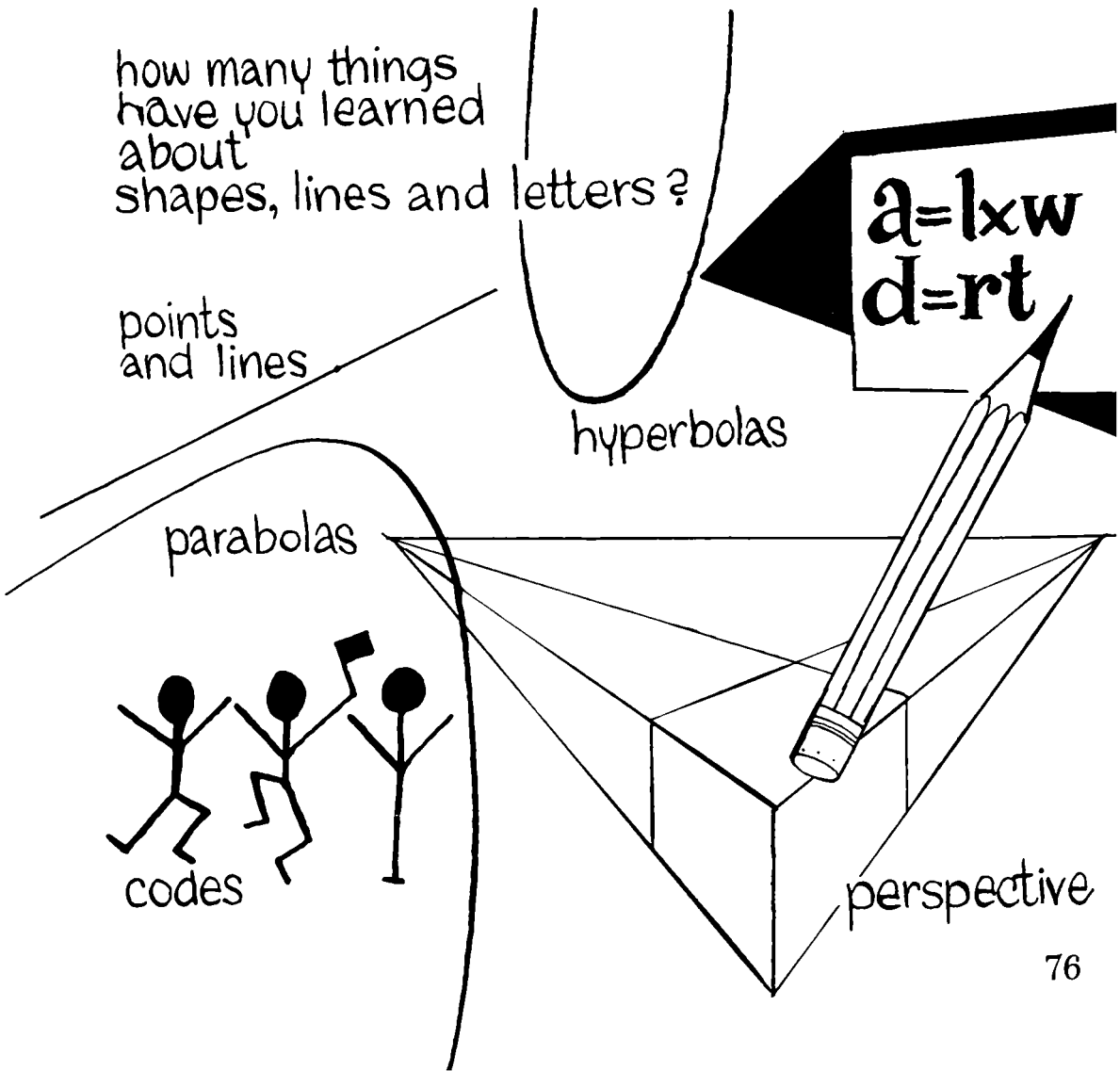


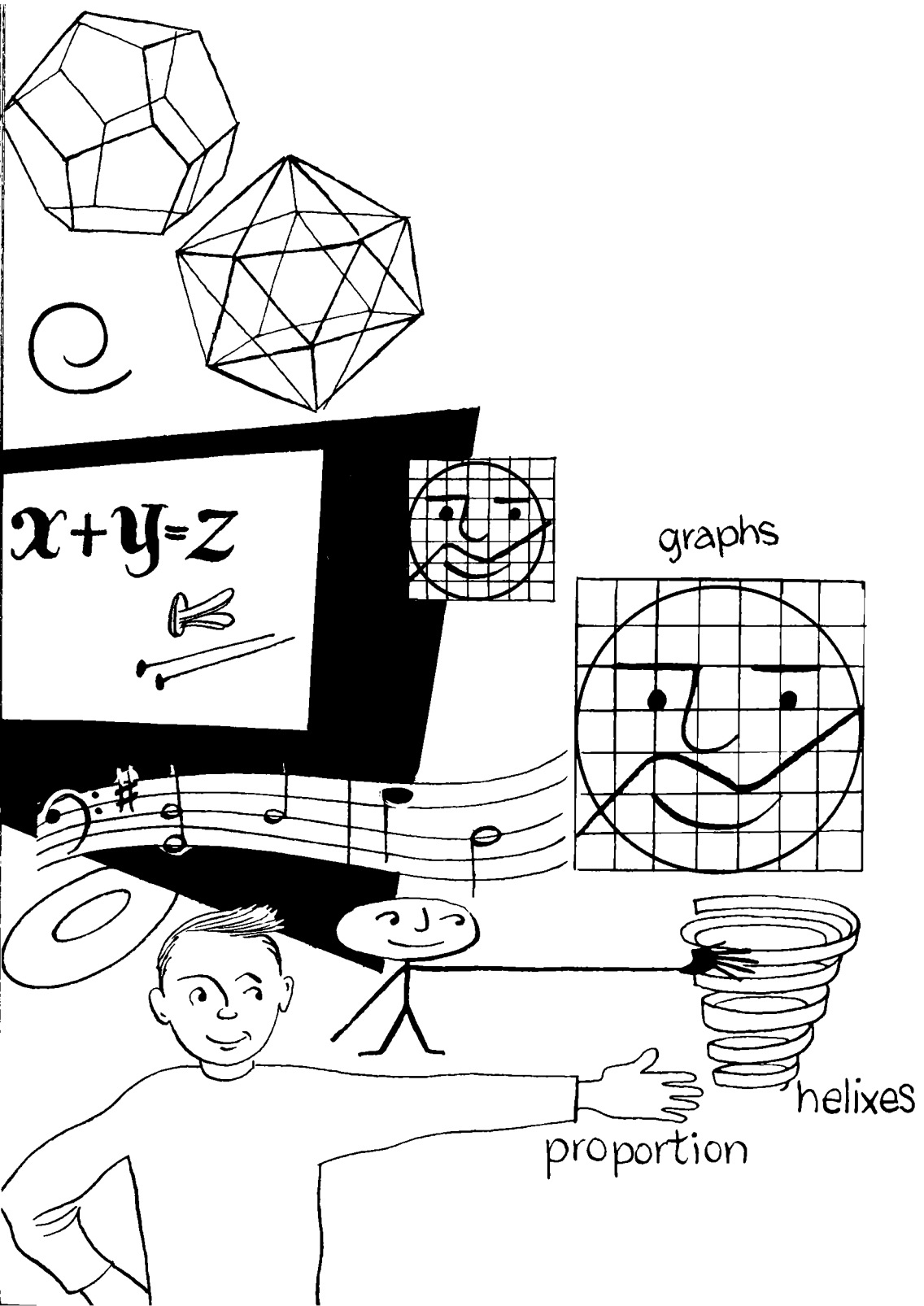


Dr. Einstein

And even the famous and far-seeing Dr. Albert Einstein could not tell where his mathematics of energy and time and space would lead.

No one can tell, yet, what wonderful things may come from our new mathematics, with numbers and without numbers. We can dream and we can imagine, but the only thing we know for *sure*—because we have learned from history, and from experience—is that when those things come, they will be more marvelous than anything we can dream, and more exciting than anything we can imagine.

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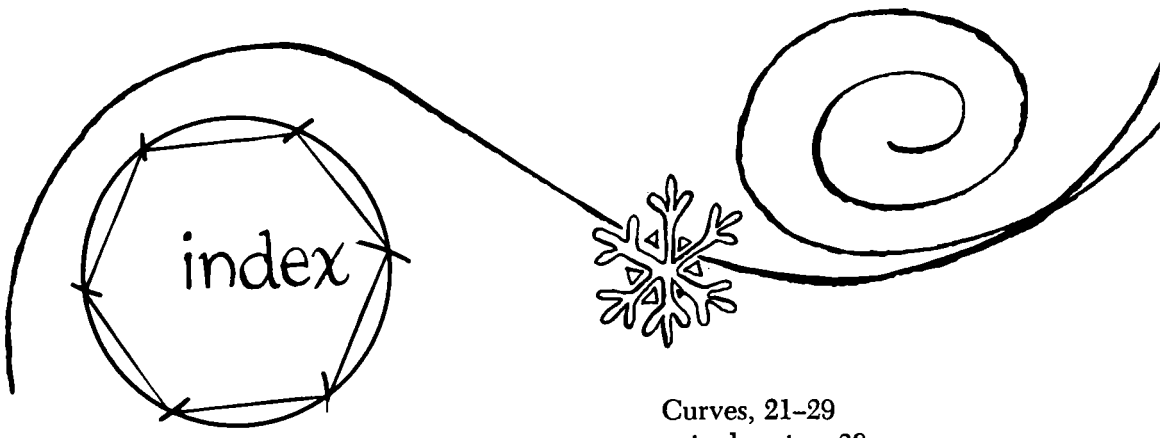
proportion



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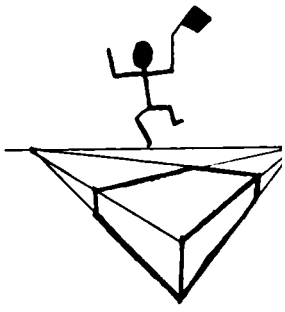

About the Authors

JEANNE BENDICK has both written and illustrated a great number of books for young people, including many in the science field.

With her clear, concise writing, and her illustrative—and amusing—drawings, she is able to make even the most difficult subject simple and exciting.

Whittlesey House books that she has written and illustrated are: *How Much and How Many,* updated 1960; *Electronics for Young People,* New Fourth Edition; *All Around You, What Could You See;* with her children, *Have a Happy Measle;* with Robert Bendick, *Television Works Like This,* Third Revised Edition; and with Marcia Levin, *Take A Number.*

Mrs. Bendick lives in Rye, New York, with her husband and two children.

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